Geometry Review

Take - Up
1. Determine the measure of each missing interior angle.

\[ \alpha + 87 + 57 = 180 \]
\[ \alpha + 144 = 180 \]
\[ \alpha = 180 - 144 \]
\[ \alpha = 36^\circ \]

\[ b + 103 + 83 + 26 = 360 \]
\[ b + 26 = 360 \]
\[ b = 360 - 262 \]
\[ b = 98^\circ \]

\[ C + 112 + 103 + 145 + 87 + 156 = 720 \]
\[ C + 603 = 720 \]
\[ C = 720 - 603 \]
\[ C = 117^\circ \]
(d) \[
\angle d + 159 + 147 + 129 + 144 + 73 + 150 = 900 \\
\angle d + 802 = 900 \\
\angle d = 900 - 802 \\
\angle d = 98^\circ
\]

(e) \[
\angle e + 137 + 85 + 141 + 97 = 540 \\
\angle e + 460 = 540 \\
\angle e = 540 - 460 \\
\angle e = 80^\circ
\]

(f) \[
\angle f + 90 + 143 + 163 + 90 + 165 = 720 \\
\angle f + 651 = 720 \\
\angle f = 720 - 651 \\
\angle f = 69^\circ
\]
2. Determine the measure of the interior angles of each figure.
   a) a regular 12-gon
   b) a regular 15-gon
   c) a regular 20-gon

\[
\begin{align*}
\text{Sum of } & = (n-2) \times 180 \\
\text{I.A.} & = \frac{\text{Sum}}{n} \\
\text{12-gon} & = 10 \times 180 \\
& = 1800 \\
\text{one I.A.} & = \frac{1800}{12} \\
& = 150^\circ \\
\text{15-gon} & = 13 \times 180 \\
\text{one I.A.} & = \frac{2340}{15} \\
& = 156^\circ \\
\text{20-gon} & = 18 \times 180 \\
\text{one I.A.} & = \frac{3240}{20} \\
& = 162^\circ
\end{align*}
\]
3. Each interior angle \( a \) in a regular \( n \)-gon has a measure of \( a = 20n \). How many sides does the polygon have?

3 sides \( \rightarrow \) I.A. = 60°

Check \( 60 = 20(3) \)

60 = 60 \( \checkmark \)

4 sides \( \rightarrow \) I.A. = 90°

Check \( 90 = 20(4) \)

90 = 80 \( \times \)

5 sides \( \rightarrow \) I.A. = 108°

Check \( 108 = 20(5) \)

108 = 100 \( \times \)

6 sides \( \rightarrow \) I.A. = 120°

Check \( 120 = 20(6) \)

120 = 120 \( \checkmark \)
4. Determine the measure of each missing angle. Support your answer with mathematical reasoning.

**a)**

Co-interior angles:

\[ \Rightarrow \text{add to } 180^\circ \]

**b)**

Using supplementary angles:

\[ b + 115 = 180 \]
\[ b = 180 - 115 \]
\[ b = 65^\circ \]

\[ a = 115^\circ \text{ (alternate "2")} \]

\[ c + 106 + 126 + 68 = 360 \]
\[ c + 300 = 360 \]
\[ c = 360 - 300 \]
\[ c = 60^\circ \]
Supplementary
\[ \angle + 92 = 180 \]
\[ \angle = 180 - 92 \]
\[ \angle = 88^\circ \]
\[ d + 63 + 88 = 180 \]
\[ d + 151 = 180 \]
\[ d = 180 - 151 \]
\[ d = 29^\circ \]

\[ \angle + 123 + 47 + 120 + 107 = 540 \]
\[ \angle + 447 = 540 \]
\[ \angle = 540 - 447 \]
\[ \angle = 93^\circ \]

\[ e + 93 = 180 \]
\[ e = 180 - 93 \]
\[ e = 87^\circ \]
6. Complete the table for each regular polygon.

<table>
<thead>
<tr>
<th>Figure</th>
<th>Measure of Each Interior Angle</th>
<th>Measure of Each Exterior Angle</th>
<th>Sum of Interior Angles</th>
<th>Sum of Exterior Angles</th>
</tr>
</thead>
<tbody>
<tr>
<td>C</td>
<td>90°</td>
<td>90°</td>
<td>360°</td>
<td>360°</td>
</tr>
<tr>
<td>D</td>
<td>108°</td>
<td>72°</td>
<td>540°</td>
<td>360°</td>
</tr>
<tr>
<td>E</td>
<td>120°</td>
<td>60°</td>
<td>720°</td>
<td>360°</td>
</tr>
<tr>
<td>B</td>
<td>135°</td>
<td>45°</td>
<td>1080°</td>
<td>360°</td>
</tr>
</tbody>
</table>

\[
\text{Sum of I.A.} = (n-2) \times 180
\]
\[
\text{one I.A.} = \frac{\text{sum of I.A.}}{n}
\]
\[
\text{one E.A.} = 180 - \text{one I.A.} \left(\frac{\text{OR}}{360}\right)
\]
\[
\text{sum of E.A.} = 360° \left(\text{one E.A.} \times n\right)
\]
7. Determine the measure of each missing angle.

a) \( \angle BDC = 30^\circ \)

   Alternate angle (\( \angle Z \))

   DE bisects \( \angle BDC \)

   \( \angle EDC = \frac{\angle BDC}{2} \)

   \( = \frac{30}{2} = 15^\circ \)

   Isosceles \( \triangle \)

   \( \angle A = 15^\circ \)

   \( x + 15 + 15 = 180 \)

   \( x + 30 = 180 \)

   \( x = 180 - 30 \)

   \( x = 150^\circ \)

   Note: Equal sides meet at the “odd” angle. The other two are the equal angles.
Supplementary
\[ \Delta + 120 = 180 \]
\[ \Delta = 180 - 120 \]
\[ \Delta = 60^\circ \]

Angles in a \( \Delta \)
\[ \Delta + 20 + 60 = 180 \]
\[ \Delta + 80 = 180 \]
\[ \Delta = 180 - 80 \]
\[ \Delta = 100^\circ \]

Supplementary
\[ \Delta + 100 = 180 \]
\[ \Delta = 180 - 100 \]
\[ \Delta = 80^\circ \]

Isosceles \( \Delta \)
\[\Rightarrow \Delta = 80^\circ \]

Angles in \( \Delta \)
\[\Rightarrow x + 80 + 80 = 180 \]
\[ x + 160 = 180 \]
\[ x = 180 - 160 \]
\[ x = 20^\circ \]

Regular octagon
\[ E.A. = \frac{360}{n} \]
\[ y = \frac{360}{8} \]
\[ y = 45^\circ \]

I.A. = 180 - E.A.
\[ x = 180 - 45 \]
\[ x = 135^\circ \]
1. Calculate the missing angle in each case.

a) \[ a + 109 + 82 + 37 = 360 \]
\[ a + 228 = 360 \]
\[ a = 360 - 228 \]
\[ a = 132^\circ \]

b) \[ b + 135 + 93 + 159 + 102 = 540 \]
\[ b + 491 = 540 \]
\[ b = 540 - 491 \]
\[ b = 49^\circ \]
2. Bob claims that the sum of the interior angles of a regular octagon is 900°. Is he correct? Justify your decision.

\[
\text{sum of I.A.} = (n-2) \times 180 \\
= (8-2) \times 180 \\
= 6 \times 180 \\
= 1080°
\]

No, Bob is incorrect. The sum of the I.A. for a regular octagon is 1080°.
3. The formula for calculating the sum of the interior angles of any $n$-gon is $(n - 2) \times 180^\circ$.

a) Explain why 2 is subtracted from $n$.

b) Explain why $(n - 2)$ is multiplied by $180^\circ$.

a) Subtract 2 because the non-intersecting diagonals create two less triangles than side lengths.

b) Multiply by $180^\circ$ because there is $180^\circ$ in one triangle.
4. a) Calculate the measure of each interior angle of a regular 25-gon.

b) What is the measure of each exterior angle?

\[
\text{Sum of I.A.} = (n-2) \times 180 = (25-2) \times 180 = 23 \times 180 = 4140 \\
\text{One I.A.} = \frac{4140}{25} = 165.6^\circ
\]

\[
\text{One E.A.} = 180 - \text{I.A.} = 180 - 165.6^\circ = 14.4^\circ
\]
5. Find the value of each unknown.

a) \[ \text{Sum of E.A.} = 360 \]
\[ x + 120 + 85 = 360 \]
\[ x + 205 = 360 \]
\[ x = 360 - 205 \]
\[ x = 155^\circ \]

b) \[ \text{Heptagon} \]
\[ \text{Sum} = (n-2) \times 180 \]
\[ \text{of I.A.} \]
\[ = (7-2) \times 180 \]
\[ = 5 \times 180 \]
\[ = 900^\circ \]
\[ \text{one I.A.} = \frac{\text{Sum of I.A.}}{n} \]
\[ = \frac{900}{7} \]
\[ = 128.57^\circ \]
Supplementary

\[ \angle A + 162 = 180 \]
\[ \angle A = 180 - 162 \]
\[ \angle A = 18^\circ \]

\[ x + 65 + 18 = 180 \]
\[ x + 83 = 180 \]
\[ x = 180 - 83 \]
\[ x = 97^\circ \]
6. Calculate the value of $x$ in each case.

a) $\Delta + 145 = 180$
   $\Delta = 180 - 145$
   $\Delta = 35^\circ$

   $x + (x-25) + 35 = 180$
   $2x - 25 + 35 = 180$
   $2x + 10 = 180$
   $2x = 170$
   $\frac{2x}{2} = \frac{170}{2}$
   $x = 85^\circ$

b) $x + x + (x-35) + (2x - 75) = 360$

   $5x - 110 = 360$
   $5x = 360 + 110$
   $5x = 470$
   $\frac{5x}{5} = \frac{470}{5}$
   $x = 94^\circ$
1. What is the relationship between the number of sides of a polygon and the number of diagonals that can be drawn from one vertex?

\[ \text{# of diagonals} = \text{# of sides} - 2 \]
12. Asad designed a tabletop in the shape of a regular pentagon. His teacher suggested he redesign it as a regular hexagon. By how much would each interior angle change?

A. $12^\circ$  B. $30^\circ$  C. $36^\circ$  D. $180^\circ$

**Regular pentagon**

\[
\text{Sum of } = (n-2) \times 180 \\
\text{I.A. } = (5-2) \times 180 \\
= 3 \times 180 \\
= 540 \\
\text{one I.A. } = \frac{540}{5} \\
= 108^\circ
\]

**Regular hexagon**

\[
\text{Sum of } = (n-2) \times 180 \\
\text{I.A. } = (6-2) \times 180 \\
= 4 \times 180 \\
= 720^\circ \\
\text{one I.A. } = \frac{720}{6} \\
= 120^\circ
\]

\[\text{Change } = 120^\circ - 108^\circ = 12^\circ \Rightarrow \boxed{A}\]
13. Determine the missing angles in each case.

a) \[ \alpha + 83 + 62 = 180 \]
   \[ \alpha + 145 = 180 \]
   \[ \alpha = 180 - 145 \]
   \[ \alpha = 35^\circ \]

b) \[ C + 53 = 180 \]
   \[ C = 180 - 53 \]
   \[ C = 127^\circ \]

\[ d + 127 + 114 = 540 \]
\[ + 92 + 119 \]
\[ d + 452 = 540 \]
\[ d = 540 - 452 \]
\[ d = 88^\circ \]

\[ d + e = 180 \]
\[ 88 + e = 180 \]
\[ e = 180 - 88 \]
\[ e = 92^\circ \]