Solve Linear Systems by Elimination

Learning Goals

- I know what the elimination method is
- I know how to add/subtract equations
- I know how to multiply a linear equation by a constant

Vocabulary

**Elimination method:**
- an algebraic method of solving a system of linear equations
- the equations are added or subtracted to eliminate one variable

Steps to Solve

1. Write the equations so that like terms appear in a column.
2. You want to have the same size of the coefficients for one of the variables.
3. If you don't, you need to multiply one of the equations by a constant so that you do.
4. If the signs of the coefficients are the same, subtract the equations; if they are different add them.
5. Solve the resulting equation.
6. Substitute this solved value into the original (other) equation and solve to find the value of the other variable. You have found the solution.
7. Check to see that the two values work in BOTH equations.
Activity
Add or Subtract Linear Equations
Work in a group of four.

1. a) Each person chooses a different linear system.
   System A  
   \[ x - y = 1 \]  ①
   \[ 2x + 5y = 16 \]  ②
   System C  
   \[ x - y = 2 \]  ①
   \[ 2x + 3y = 14 \]  ②
   System B  
   \[ x - 3y = 6 \]  ①
   \[ 3x + 4y = -21 \]  ②
   System D  
   \[ x + y = 5 \]  ①
   \[ 2x - 5y = 17 \]  ②

   b) On grid paper, graph the linear system. Label each equation. What is the point of intersection?

2. a) Add the equations in your linear system. Label the resulting equation ③.
   b) Graph equation ③ on the same set of axes as you used in question 1. What do you notice?

3. a) Subtract one equation from the other. Label the resulting equation ④.
   b) Graph equation ④ on the same set of axes as you used in questions 1 and 2. What do you notice?

6. Reflect  Make a statement about the equation that results from adding or subtracting the equations in a linear system.

Multiply a Linear Equation by a Constant
Work in a group of four.

1. a) Each person chooses a different linear equation.
   Equation A  
   \[ y = 4x + 3 \]
   Equation C  
   \[ y = -x - 2 \]
   Equation B  
   \[ 2x + y = 5 \]
   Equation D  
   \[ 3x - y = 1 \]

   b) Use grid paper. Graph the line and label it with its equation.

2. a) Choose a number between -5 and 5. Multiply each term in your equation by the number you chose.
   b) Graph the resulting equation on the same set of axes. What do you notice?
   c) Choose a different number. Multiply each term in your original equation by the number and graph the result on the same set of axes.

6. Reflect  Make a statement about the equation that results from multiplying each term in an equation by a constant.
**Solve by Elimination**

Use the elimination method to solve the linear system

\[ 3x + y = 19 \quad \text{and} \quad 4x - y = 2. \]

Coefficients of \( y \) are the same.

\[ 3x + y = 19 \]
\[ 4x - y = 2 \]

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\[
\frac{7x}{7} = \frac{21}{7}
\]
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\( x = 3 \)

Check in \( 2 \)

\[ 4x - y = 2 \]
\[ 4(3) - (10) = 2 \]
\[ 12 - 10 = 2 \quad \checkmark \]

Sub into \( 1 \)

\[ 3x + y = 19 \]
\[ 3(3) + y = 19 \]
\[ 9 + y = 19 \]
\[ y = 19 - 9 \]
\[ y = 10 \]

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**Make a Coffee Blend**

Jemma is making 120 kg of a new blend of coffee that sells for $18/kg. The blend is made from two kinds of coffee: one that sells for $10/kg, and another that sells for $15/kg. How much of each type of coffee should Jemma use to make the new blend?

First write the equations.

Let \( x \) = $18 coffee, \( y \) = $10 coffee

\[ x + y = 120 \]

Total amount of coffee sold.

\[ 18x + 10y = 15(120) \]
\[ 18x + 10y = 1800 \]

Cost of coffee blend.

\( x \) \times 10 to be able to eliminate terms.

\( 10x + 10y = 1200 \)

\( x + y = 120 \)

Sub into \( 1 \)

\[ x + y = 120 \]
\[ (75) + y = 120 \]
\[ y = 120 - 75 \]
\[ y = 45 \]

Need 75 kg of $18 coffee and 45 kg of $10 coffee.

Check in \( 2 \)

\[ 18x + 10y = 1800 \]
\[ 18(75) + 10(45) = 1800 \]
\[ 1350 + 450 = 1800 \quad \checkmark \]
Questions

1. Use the elimination method to solve each linear system.
   a) \( x + y = 2 \)
   \( 3x - y = 2 \)
   b) \( x - y = -1 \)
   \( 3x + y = -7 \)
   c) \( 2x + y = 8 \)
   \( 4x - y = 4 \)
   b) \( x - y = -6 \)
   \( 2x + y = -6 \)
   d) \( 2x + y = -5 \)
   \( -2x + y = -1 \)
   f) \( 4x - y = -1 \)
   \( -4x - 3y = -15 \)

4. Solve each linear system.
   a) \( 4x + 3y = 4 \)
   \( 8x - y = 1 \)
   b) \( 5x - 3y = 2 \)
   \( 10x + 3y = 5 \)
   c) \( 5x + 2y = 48 \)
   \( x + y = 15 \)
   d) \( 2x + 3y = 8 \)
   \( x - 2y = -3 \)

5. Abby mixes cinnamon and nutmeg to make 25 g of a spice mix. Cinnamon costs $6/g and nutmeg costs $12.50/g. The spice mix costs $9.75/g. How much of each spice does Abby need to use?

6. Tickets for a play cost $5 for adults and $3 for children. A total of 800 tickets are sold and total sales are $3600.
   a) Write a system of linear equations to represent the situation.
   b) How many adult tickets are sold?

7. When flying into the wind, an airplane travels at an average speed of 540 km/h. When flying with the wind, the airplane travels at an average speed of 680 km/h. Let \( z \) represent the speed of the airplane with no wind and \( w \) represent the wind speed.
   a) Write a system of linear equations to represent the situation.
   b) Describe how you would calculate the wind speed.
Solutions

1. Use the elimination method to solve each linear system:
   a) \(3x + y = 2\)
   b) \(x - y = -1\)
   c) \(2x + y = 8\)
   d) \(2x - y = -6\)
   e) \(3x - y = -5\)
   f) \(4x - y = -1\)

Add or subtract as necessary to eliminate the x or y terms

Substitute \(x = 2\):

\[x + y = 1\]
\[1 + y = 2\]
\[y = 1\]

Check in e:
\[3x - y = 2\]
\[3(1) - 1 = 2\]
\[3 - 1 = 2\]
1. Use the elimination method to solve each linear system.

   a) \( x + y = 2 \)
   \( 3x - y = 2 \)

   b) \( x - y = -1 \)
   \( 3x + y = -7 \)

   c) \( 2x + y = 8 \)
   \( 4x - y = 4 \)

   d) \( 2x - y = -6 \)
   \( 4x + y = 6 \)

   e) \( 3x + y = 5 \)
   \( -2x + y = -1 \)

   f) \( 4x - y = -1 \)
   \( -4x + 3y = -19 \)

Add or subtract as necessary to ELIMINATE the \( x \) or \( y \) terms

\( 2x + y = 8 \)
\[ \frac{6x}{6} = \frac{12}{6} \]
\[ x = 2 \]

Sub into \( 1 \)
\[ 2x + y = 8 \]
\[ 2(2) + y = 8 \]
\[ 4 + y = 8 \]
\[ y = 4 \]

Check in \( 2 \)
\[ 4x - y = 4 \]
\[ 4(2) - (4) = 4 \]
\[ 8 - 4 = 4 \checkmark \]

\( 2x - y = -6 \)
\[ \frac{6x}{6} = \frac{-12}{6} \]
\[ x = -2 \]

Sub into \( 1 \)
\[ 2x - y = -6 \]
\[ 2(-2) - y = -6 \]
\[ -4 - y = -6 \]
\[ -y = 2 \]
\[ y = -2 \]

Check in \( 2 \)
\[ 4x + y = -6 \]
\[ 4(-2) + (2) = -6 \]
\[ -8 + 2 = -6 \]

\( 2x + y = -5 \)
\[ -2x + y = -1 \]

\[ \frac{4x}{4} = \frac{-4}{4} \]
\[ x = -1 \]

Sub into \( 1 \)
\[ 2x + y = -5 \]
\[ 2(-1) + y = -5 \]
\[ -2 + y = -5 \]
\[ y = -3 \]

Check in \( 2 \)
\[ -2x + y = -1 \]
\[ -2(-1) + (-3) = -1 \]
\[ 2 - 3 = -1 \]

\[ \frac{4x - y = -1}{-4x + 3y = -19} \]
\[ 4x - (5) = -1 \]
\[ 4x - 5 = -1 \]
\[ 4x = 4 \]
\[ x = 1 \]

Check in \( 2 \)
\[ -4x + 3y = -19 \]
\[ -4(1) - 3(5) = -19 \]
\[ -4 - 15 = -19 \]
Multiply an equation by a constant so that when you add or subtract you do eliminate a variable.

4. Solve each linear system.
   a) $4x + 3y = 4$
      $8x - y = 1$
id) $5x + 2y = 48$
   $x + y = 15$

   Multiply an equation by a constant so that when you add or subtract you do eliminate a variable.

   a) $4x + 3y = 4$
      $8x - y = 1$

   Multiply an equation by a constant so that when you add or subtract you do eliminate a variable.

   a) $4x + 3y = 4$
      $8x - y = 1$

   Multiply an equation by a constant so that when you add or subtract you do eliminate a variable.

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   Multiply an equation by a constant so that when you add or subtract you do eliminate a variable.

   a) $4x + 3y = 4$
      $8x - y = 1$
1. Abby mixes cinnamon and nutmeg to make 25 g of a spice mix. Cinnamon costs $9/g and nutmeg costs $12.50/g. The spice mix costs $9.75/g. How much of each spice does Abby need to use?

Let \( C \) = mass of cinnamon
\( N \) = mass of nutmeg

\[
\begin{align*}
(1) \quad C + N &= 25 \\
9C + 12.5N &= 9.7(25) \\
(2) \quad 9C + 12.5N &= 242.5
\end{align*}
\]

\[ \begin{align*}
\text{Sub into (1)} \\
C + N &= 25 \\
C + (5) &= 25 \\
C &= 25 - 5 \\
\text{Check in (2)} \\
9C + 12.5N &= 242.5 \\
9(20) + 12.5(5) &= 242.5 \\
180 + 62.5 &= 242.5 \\
\]

Abby needs to use 5 g of nutmeg and 20 g of cinnamon.

2. Tickets for a play cost $3 for adults and $3 for children. A total of 800 tickets are sold and total sales are $3000.

Let \( a \) = # adults
\( c \) = # children

\[
\begin{align*}
(1) \quad a + c &= 800 \\
5a + 3c &= 3600 \\
\times 3 \Rightarrow \\
3a + 3c &= 2400
\end{align*}
\]

\[ \begin{align*}
\text{Sub into (1)} \\
a + c &= 800 \\
(600) + c &= 800 \\
c &= 800 - 600 \\
\text{Check in (2)} \\
a + c &= 800 \\
5a + 3c &= 3600 \\
5(600) + 3(200) &= 3600 \\
3000 + 600 &= 3600 \\
\]

They sold a total of 600 adult tickets.
7. When flying into the wind, an airplane travels at an average speed of 540 km/h. When flying with the wind, the airplane travels at an average speed of 680 km/h. Let $s$ represent the speed of the airplane with no wind and $w$ represent the wind speed.

a) Write a system of linear equations to represent the situation.

b) Describe how you would calculate the wind speed.

\[
\begin{align*}
(a) \quad s - w &= 540 \quad \text{(1)} \\
     s + w &= 680 \quad \text{(2)}
\end{align*}
\]

(b) To calculate the wind speed, add the equations and solve for $s$.

Use this value in equation (2) to solve for $w$. Check both values work in (1).

\[
\begin{align*}
(\text{1)} + \text{(2)}: \quad 2s &= 1220 \\
\frac{2s}{2} &= \frac{1220}{2} \\
s &= 610 \text{ km/h}
\end{align*}
\]

Sub into (2):

\[
\begin{align*}
s + w &= 680 \\
610 + w &= 680 \\
w &= 680 - 610 \\
w &= 70 \text{ km/h}
\end{align*}
\]

Check in (1):

\[
\begin{align*}
s - w &= 540 \\
610 - 70 &= 540
\end{align*}
\]