Model Problems with Equations

15. The formula \( R = 9T - 70 \) models the chirping rates of crickets at various temperatures. The variable \( R \) represents the mean number of chirps per minute, and \( T \) represents the temperature, in degrees Celsius.

a) When the rate is 11 chirps/minute, what is the approximate temperature?

\[
ll = 9T - 70
\]

\[
ll + 70 = 9T - 70 + 70
\]

\[
\frac{81}{9} = \frac{9T}{9}
\]

\[
q = T
\]

Temperature is 9°C

b) \( R = 9T - 70 \)

\[
R = 9(20) - 70
\]

\[
R = 180 - 70
\]

\[
R = 110
\]

\[\Rightarrow\] 110 chirps/minute at 20°C

Natalie is helping her mother decorate a quilt. They sew one piece of ribbon along each side of the equilateral triangles in a pattern.

1. Model the pattern using toothpicks.

2. How many pieces of ribbon are needed for one triangle? two triangles? three triangles? Organize the information.

<table>
<thead>
<tr>
<th># Δs</th>
<th># Ribbons</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3</td>
</tr>
<tr>
<td>\frac{2}{3}</td>
<td>\frac{3}{5}</td>
</tr>
<tr>
<td>\frac{3}{7}</td>
<td></td>
</tr>
</tbody>
</table>

3. Describe the pattern.

For each extra triangle we need 2 more ribbons
4. Model the pattern using a formula.

\[ R = 2n + 1 \]

5. Verify your formula. Is your formula correct? If not, revise it.

Test for \( n = 3 \)

\[ 2(3) + 1 = ? \]

6. Natalie uses 65 pieces of ribbon to make a string of triangles. How many triangles does she sew?

\[ R = 2n + 1 \]

\[ 65 = 2n + 1 \]

\[ 64 = 2n \]

\[ n = \frac{64}{2} \]

\[ n = 32 \]

There will be 32 triangles.

Example 1: Solve a Patterning Problem

Juan measures the heights of stacks of cups to be 8.5 cm, 10 cm, and 11.5 cm.

a) Describe the pattern.

b) Predict the height of the next three stacks of cups.

c) Model the pattern with a formula. Explain what your formula means.

d) You have a stack of 100 cups. Use your formula to find the height.

e) Juan measures a stack to be 52 cm high. How many cups are in the stack?

\[ H = 1.5n + 7 \]

\[ H = 1.5(100) + 7 \]

\[ H = 150 + 7 \]

\[ H = 157 \text{ cm} \]

\[ \frac{52}{1.5} = \frac{1.5n}{1.5} \]

\[ 30 = n \]

\[ \Rightarrow \text{30 cups in the stack!} \]
Example 2: Solve a Problem Using an Equation

Maria has a mould of a square candle dish. Each side length is 10 cm. She wants her new square candle dish to have a perimeter of 60 cm. By how much does Maria have to increase each side length of her mould?

\[ P = 4x \]

\[ x = \text{side length} \]

\[ P = \text{perimeter} \]

\[ P = 4x \]

\[ 60 = \frac{4x}{4} \]

\[ \frac{60}{4} = x \]

\[ 15 = x \]

Increase = 15 - 10

= 5 cm on each side.

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**Key Ideas**

Problems can be modelled and solved using equations.

To develop an equation,

- Draw a diagram or make a table.
- Identify the variables.
- Look for a pattern.
- Translate the information using numbers and operations.

**Note:**

\[ H = 7 + 1.5c \]

is the same as

\[ H = 1.5c + 7 \]

Deal with the +7 first.

The perimeter of a rectangular frame is 30 cm. This is 14 cm more than the perimeter of the mirror. What is the length of one side of the mirror?

Let \( x \) represent the length of one side of the square mirror.

\[ 30 = 4x + 14 \]

\[ 30 - 14 = 4x + 14 - 14 \]

\[ 16 = 4x \]

\[ 16 ÷ 4 = 4x ÷ 4 \]

\[ 4 = x \]

The length of one side of the mirror is 4 cm.