

MTH1W Grade 9 Mathematics

6.8 Proportional Relationships and Conversions

- Goal(s)**
- To explain the meaning of a proportional relationship
 - To solve problems by writing and solving proportions
 - To determine if a linear relationship is an example of direct or partial variation

Jun 19-8:29 AM

A car is travelling at a constant speed of 115 km/h.

How far will the car travel in 1.5 hours? 3 hours? 10 hours?

115 km in 1 hour

x km in 1.5 hours

$$\Rightarrow x = 115 \times 1.5$$

$$x = 172.5 \text{ km}$$

y km in 3 hours

$$\Rightarrow y = 115 \times 3$$

$$y = 345 \text{ km}$$

z km in 10 hours

$$\Rightarrow z = 115 \times 10$$

$$z = 1150 \text{ km}$$

115 x time

In **proportional relationships**, the quantities involved are **related** by **multiplication** or **division**.

The statement, $5 : 60 = 35 : d$ is a **proportion**. A **proportion** is a statement that *two ratios are equal*.

Proportions can also be represented as equivalent fractions:

$$\frac{5}{60} = \frac{35}{d}$$

If a proportion contains an unknown, the value can be determined by **inspection** or through the process of **cross-multiplication**.

$$\frac{5}{60} = \frac{35}{d}$$

x7

"If five times seven is thirty-five, then d is equal to sixty multiplied by seven..."

$$\therefore d = 420!"$$

$$\frac{5}{60} = \frac{35}{d}$$

$$(5)(d) = (60)(35)$$

$$5d = 2100$$

$$\frac{5d}{5} = \frac{2100}{5}$$

$$d = 420$$

Determine the value of each unknown.

$$\frac{6}{7} = \frac{24}{w}$$

$\xrightarrow{\times 4}$

$$\Rightarrow 7 \times 4 = w$$

$$28 = w$$

$$\frac{-5}{15} = \frac{1}{g}$$

$\xleftarrow{\times -5}$

$$\Rightarrow \frac{15}{-5} = \frac{-5g}{-5}$$

$$-3 = g$$

$$\frac{4.7}{m} = \frac{164.8}{41.3}$$

Cross-multiply

$$4.7(41.3) = 164.8m$$

$$\frac{194.11}{164.8} = \frac{164.8m}{164.8}$$

$$1.18 = m$$

A gear ratio is the ratio of the number of teeth in two connected gears.

The gear ratio of two gears is 3 : 2.

a) If the larger gear has 126 teeth, how many teeth does the smaller gear have?

b) If the smaller gear has 126 teeth, how many teeth does the larger gear have?

Use your knowledge of proportional relationships to solve these problems.

$$a) \frac{\text{large}}{\text{small}}$$

$$\Rightarrow \frac{126}{n} = \frac{3}{2}$$

$$126(2) = 3n$$

$$\frac{252}{3} = \frac{3n}{3}$$

$$84 = n$$

$$\Rightarrow 84 \text{ teeth}$$

$$b) \frac{\text{large}}{\text{small}}$$

$$\Rightarrow \frac{n}{126} = \frac{3}{2}$$

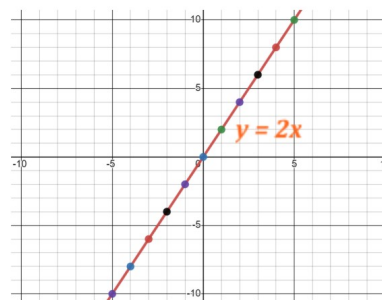
$$2n = 3(126)$$

$$\frac{2n}{2} = \frac{378}{2}$$

$$n = 189$$

$$\Rightarrow 189 \text{ teeth}$$

When graphed, a **proportional relationship** is linear, and the line has an **initial value of 0** (graph passes through the origin). A proportional relationship is an example of **direct variation**. The equation of a direct variation relationship is $y = mx$.



A linear relation that is **not proportional**, has an **initial value other than 0** (does not pass through the origin). A linear, non-proportional relationship is an example of **partial variation**. The equation of a direct variation relationship is $y = mx + b$.

