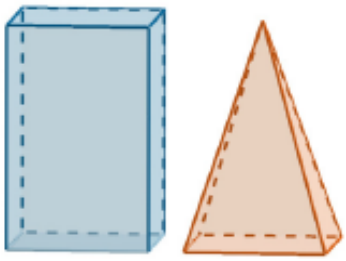
Solutions

Page 294 #s 1 – 6, 12, 13, 19, 20

1. Describe the relationship between the volume of a prism and the volume of a pyramid with the same base shape and same height.

The volume of a pyramid will be one third of the volume of a prism with the same base shape and same height.

2. In the diagram on the right, the volume of the rectangular prism is 90 cm^3. Determine the volume of the pyramid (both solids have the same base and height).

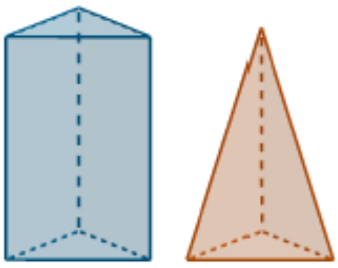


Vol pyramid $= \frac{1}{3}$ Vol prism

$= \frac{1}{3} \times 90$

$= 30 cm^3$

3. In the diagram on the right, the pyramid has a volume of 12.5 m^3. Determine the volume of the prism (both solids have the same base and height).



Vol prism $= 3 \times$ Vol pyramid

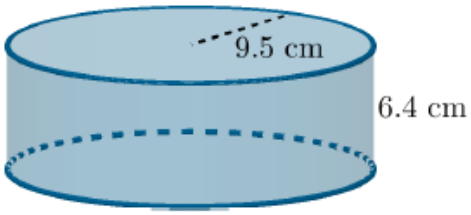
$= 3 \times 12.5$

$= 37.5 m^3$

4. Describe the relationship between the volume of a cylinder and the volume of a cone with the same base circle and height.

The volume of a cone will be one third of the volume of a cylinder with the same base circle and height.

5. The volume of the cylinder shown on the right is 1815 cm^3. Use this information to find the volume of a cone with a radius of 9.5 cm and a height of 6.4 cm.

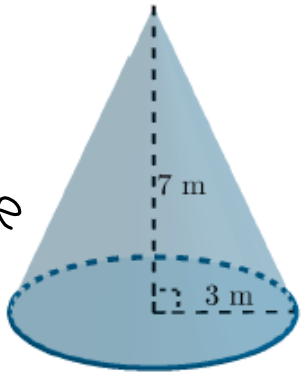


$$\text{Vol of cone} = \frac{1}{3} \text{ of Vol of cylinder}$$

$$= \frac{1}{3} \times 1815$$

$$= 605\ cm^3$$

6. The volume of the cone shown on the right is 66 m^3. Determine the volume of the cylinder that has the same base and height as this cone.

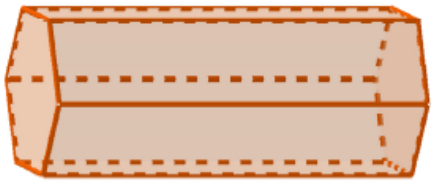


Vol of cylinder = 3 x Vol of cone

$= 3 \times 66$

$= 198\ m^3$

12. The shape of the box for a chocolate treat is a hexagonal prism, as shown on the right. Each box has a volume of 227 cm^3. For Christmas, the company will switch to a festive box in the form of a hexagon-based pyramid. This festive box, which is intended to be stood upright and portray a Christmas tree, will have the same base and height as the standard prism packaging.



a) Determine the volume of the festive box. Round your answer to two decimal places.

b) If the treat usually costs $6.99, what would be a reasonable price for the festive version?

a) Vol pyramid = $\frac{1}{3}$ x Vol of prism

$= \frac{1}{3} \times 227$

$= 75.67\ cm^3$

b) If the volume is $\frac{1}{3}$, a fair price would be $\frac{1}{3} \times \$6.99 = \$2.33$

13. The formula for the volume of a cylinder with radius r and height h is $V = \pi r^2 h$.

The volume of a cone with radius r and height h is given by $V = \frac{1}{3}\pi r^2 h$ or $V = \frac{\pi r^2 h}{3}$.

a) Explain how the relationship between the volumes of cylinders and cones is evident in these formulas.

They are essentially the same EXCEPT for finding $\frac{1}{3}$ of it OR dividing by 3 (which is the same as finding $\frac{1}{3}$).

13. The formula for the volume of a cylinder with radius r and height h is $V = \pi r^2 h$.

The volume of a cone with radius r and height h is given by $V = \frac{1}{3}\pi r^2 h$ or $V = \frac{\pi r^2 h}{3}$.

b) A cylinder has a radius of 6 cm and a height of 14 cm. Use the formula to determine the volume of the cylinder and then use the relationship between cylinders and cones to find the volume of a cone with the same base and height.

$V_{CYL} = \pi r^2 h$

$= \pi (6)^2 (14)$

$= 1583.4 \text{ cm}^3$

$V_{CONE} = \frac{V_{CYL}}{3}$

$= \frac{1583.4}{3}$

$= 527.8 \text{ cm}^3$

13. The formula for the volume of a cylinder with radius r and height h is $V = \pi r^2 h$.

The volume of a cone with radius r and height h is given by $V = \frac{1}{3}\pi r^2 h$ or $V = \frac{\pi r^2 h}{3}$.

c) A cone has a diameter of 10.4 inches and a height of 4.5 inches. Use a formula to determine the volume of the cone and then use the relationship between cylinders and cones to find the volume of a cylinder with the same base and height.

$V_{CONE} = \frac{1}{3}\pi r^2 h$

$= \frac{1}{3} \times \pi \times \left(\frac{10.4}{2}\right)^2 \times 4.5$

$= 127.4\ cm^3$

$V_{CYL} = 3 \times V_{CONE}$

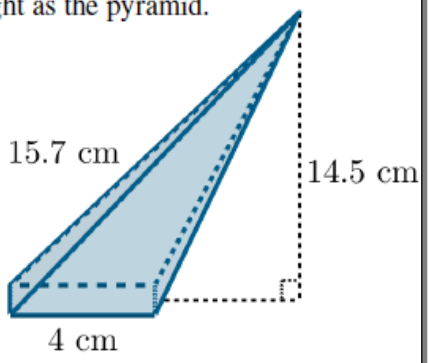
$= 3 \times 127.4$

$= 382.2\ cm^3$

19. The pyramid shown below has a square base.

a) Determine the volume of any prism that has the same base and height as the pyramid.

b) Determine the volume of the pyramid.



a) Vol = area base × height

$= (4 \times 4) \times 14.5$

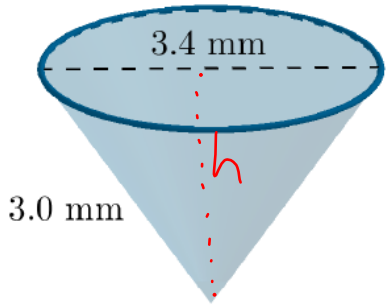
Square base

$= 232\ cm^3$

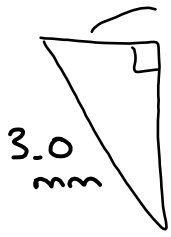
b) $Vol_{Pyr} = \frac{Vol_{prism}}{3} = \frac{232}{3}$

$= 77.3\ cm^3$

20. The inverted right cone shown below, which has a base diameter of 3.4 mm and a slant height of 3.0 mm, is inscribed in a cylinder. Determine the volume of the cylinder.



Need to calculate the vertical height "h"



$\frac{3.4}{2} = 1.7\text{ mm}$

$h^2 + 1.7^2 = 3.0^2$

$h^2 + 2.89 = 9$

$h^2 = 9 - 2.89$

$h^2 = 6.11$

$h = \sqrt{6.11}$

$h = 2.47\text{ mm}$

$Vol_{CYL} = \pi r^2 h$

$= \pi \times 1.7^2 \times 2.47$

$= 22.4\text{ mm}^3$