MTH1W Grade 9 Mathematics

6.4 Triangle and Circle Geometry

Goal(s) *- To develop an understanding of the angle and side length properties of triangles*

- To use angle relationships, chord, and tangent properties of a circle to determine unknown angles

Jun 19-8:29 AM

Triangle Geometry - Important Terms

vertex - point where two or more sides meet.

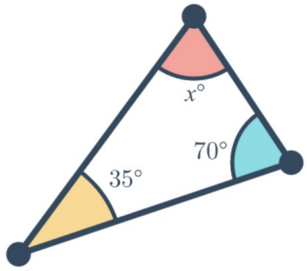
interior angle - an angle formed on the inside of a polygon by two sides meeting at a vertex.

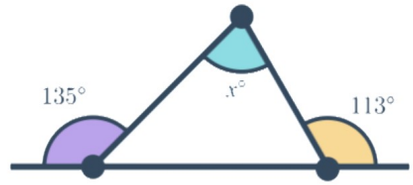
exterior angle - an angle formed on the outside of a geometric figure by extending one of the sides past the vertex.

median - a line drawn from a vertex to the midpoint of the opposite side.

altitude - a perpendicular line drawn from a vertex to the opposite side (another term to describe the height of a triangle)

Determine the unknown angle in each triangle.





$x + 35 + 70 = 180$

$x + 105 = 180$

$x + 105 - 105 = 180 - 105$

$x = 75°$

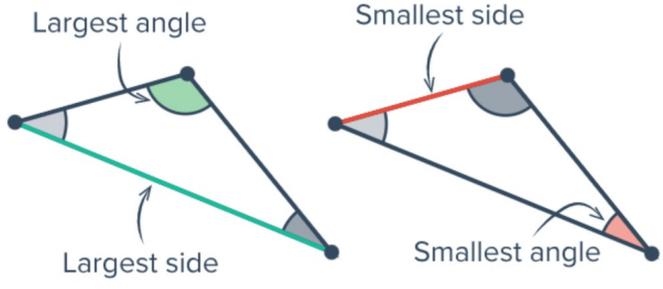
$y = 180 - 135$

$y = 55°$

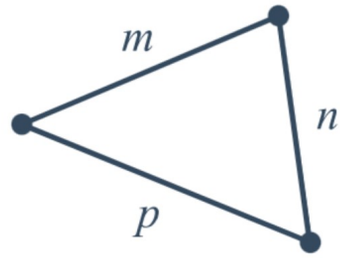
$z = 180 - 113$

$z = 67°$

In any triangle, the longest side is found across from the largest angle, and the shortest side is found across from the smallest angle.



For a triangle to exist, the combined length of each pair of sides must be longer than the remaining side.



$m + n > p$

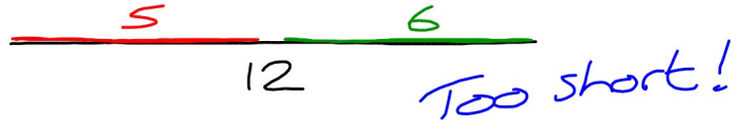
$m + p > n$

$n + p > m$

Determine whether a triangle with sides lengths of 5, 6, and 12 exist?

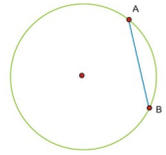
Side Length	Sum of Remaining Side Lengths
5	$6+12=18$ $18>5$ ✓
6	$5+12=17$ $17>6$ ✓
12	$5+6=11$ $11>12$ X

$\Longrightarrow$ A triangle will not exist

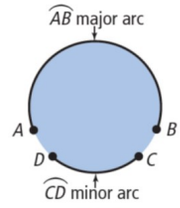


Circle Geometry - Chord Properties

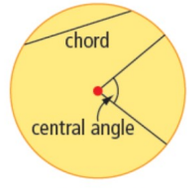
chord - A line segment joining two points on the circumference of a circle.



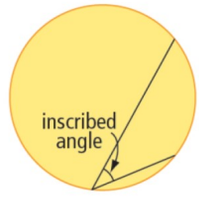
arc (of a circle) - A portion of the circumference of a circle. A minor arc is less than a semicircle, and a major arc is more than a semicircle.



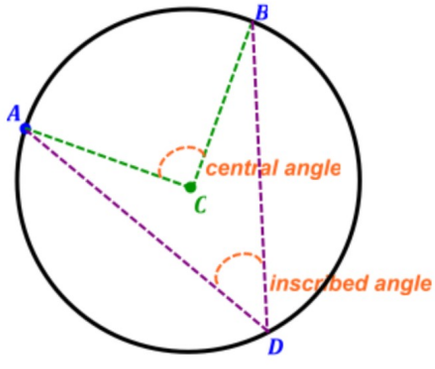
central angle - An angle formed by two radii of a circle. The vertex of the angle is at the centre of the circle, and the endpoints are on the circle.



inscribed angle - An angle formed by two chords that share a common endpoint. The vertex and endpoints are on the circle.



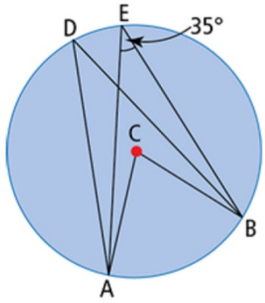
If a **central angle** and an **inscribed angle** ***lie on the same arc*** then the inscribed angle is half the size of the central angle.



central angle = $\angle ACB = 88^\circ$

inscribed angle = $\angle ADB = 44^\circ$

Point C is the centre of the circle. ∠AEB = 35°



a. What is the measure of ∠ADB? Justify your answer.

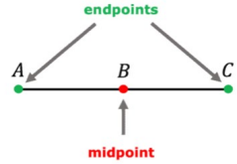
b. What is the measure of ∠ACB? Justify your answer.

a) $\angle ADB = \angle AEB = 35°$
Both angles are inscribed on the same arc.

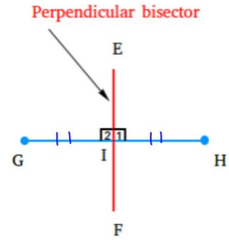
b) $\angle ACB$ = Double $\angle ADB = 2(35) = 70°$
The central angle is double the inscribed angle.

Circle Geometry - Chord Properties (cont.)

midpoint - The midpoint is the middle point of a line segment. It is equidistant from both endpoints, bisecting the segment.

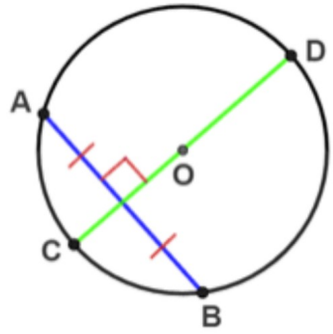


perpendicular bisector - A line or a segment perpendicular to a segment that passes through the midpoint of the segment.

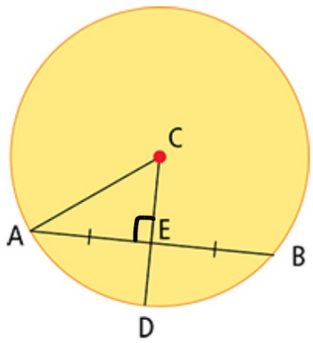


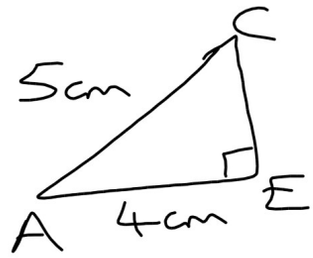
Perpendicular Bisector of a Chord

A line that passes through the centre of a circle, that bisects and is perpendicular to a chord.



Radius CD bisects chord AB. Chord AB measures 8 cm. The radius of the circle is 5 cm. What is the length of line segment CE? Justify your solution.





$AE = \frac{1}{2}AB$

$= \frac{1}{2}(8)$

$= 4cm$

$CE^2 = AC^2 - AE^2$

$CE^2 = 5^2 - 4^2$

$CE^2 = 25 - 16$

$CE^2 = 9$

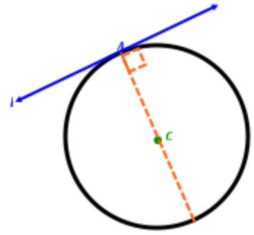
$\sqrt{CE^2} = \sqrt{9}$

$CE = 3$ $\Rightarrow CE = 3cm$

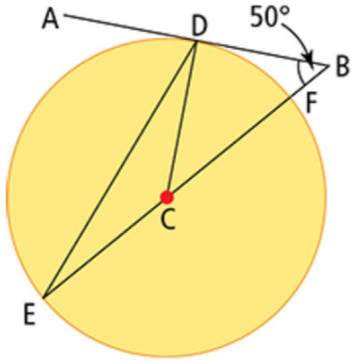
Circle Geometry - Tangent Properties

tangent to a circle - A tangent to a circle is perpendicular to the radius at the *point of tangency (where the line touches the circle)*.

tangent chord relationship - A chord drawn perpendicular to a tangent at the point of tangency contains the centre of the circle, and is a diameter.



In the diagram shown, $\boldsymbol{AB}$ is tangent to the circle at point $\boldsymbol{D}$, $\boldsymbol{BE}$ contains the diameter $\boldsymbol{FE}$, and $\angle \boldsymbol{ABE} = \mathbf{50}°$.



What is the measure of $\angle BDC$? Justify your answer.

90° – AB is tangent to radius CD

What is the measure of $\angle DCE$? Explain your reasoning.

140° – external angle theorem ($\angle CDB + \angle CBD$)

What type of triangle is ΔCDE? Explain.

Isosceles – CD = CE (both are radii)