Solutions

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1. Consider the graph of the relation $xy = 1$ shown below.

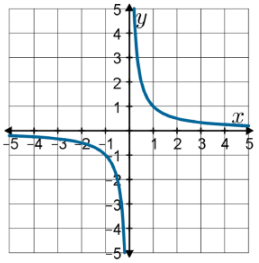
a) Explain why the graph passes through the points $(-1,-1)$ and $\left(2,\frac{1}{2}\right)$.

b) Explain why the graph does not pass through any point with an x-value or y-value of 0.

c) Explain why the graph appears in only the top right and bottom left quadrants.

d) Explain why the graph approaches the x-axis on the right-hand side.

e) Explain why the graph's y-values increase as the x-value approaches 0 from the right.



a) Because the product of the x and y values must equal one.
$(-1)\times(-1)=1$ and $2\times\frac{1}{2}=1$

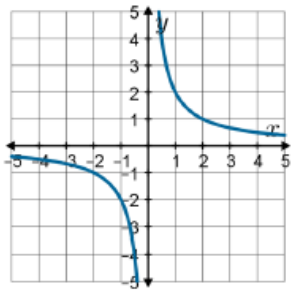
b) If x or y was zero, then the product would be zero. We want the product to equal one.

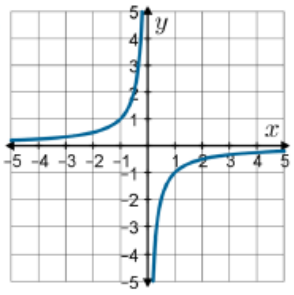
c) Product must equal positive one.
In Q1 x and y are positive
In Q3 x and y are negative
Both give a positive product
In Q2 and Q4 the products will be negative.

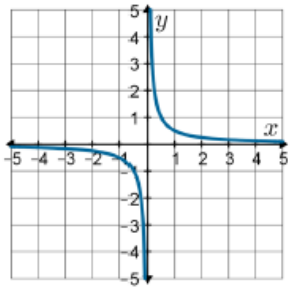
d) If the product is one, then as x gets bigger y must get smaller.

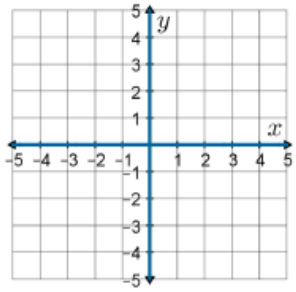
e) Similarly, as x gets smaller y must get bigger.

3. Match each graph with its corresponding equation.

a) 

b) 

c) 

d) 

i) $xy = -1$
ii) $xy = 0$
iii) $xy = 2$
iv) $xy = \frac{1}{2}$

(i) → (b)
(ii) → (d)
(iii) → (a)
(iv) → (c)

Choose a coordinate on the graph and calculate (x)(y)

4. Does the point $(2,4)$ satisfy the relation $xy = -8$? Explain.

Sub in $x = 2$ and $y = 4$

$\Rightarrow xy = -8$

$(2)(4) = -8$

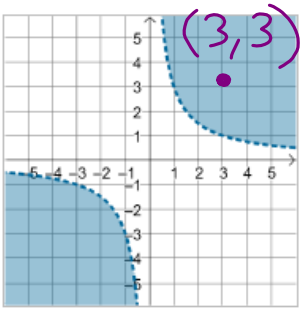
$8 \neq -8$

$\Rightarrow$ (2,4) does NOT satisfy the relation $xy = -8$

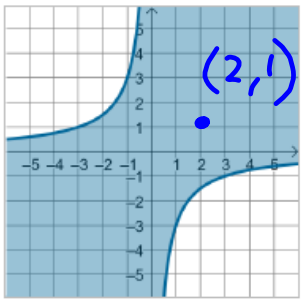
[Also, to make a negative product you need one negative and one positive value. 2 and 4 are both positive]

8. Match the graph of each inequality with its corresponding algebraic representation.

a)



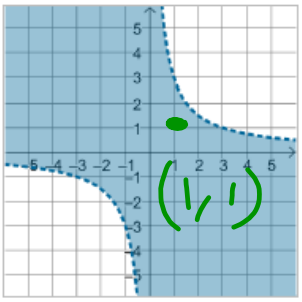
b)



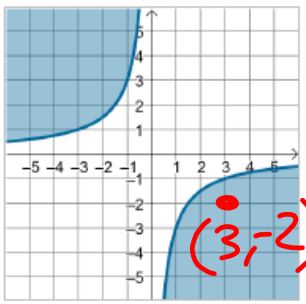
i) $xy \leq -3$
ii) $xy < 3$
iii) $xy \geq -3$
iv) $xy > 3$

(i) → (d)
(ii) → (c)
(iii) → (b)
(iv) → (a)

c)

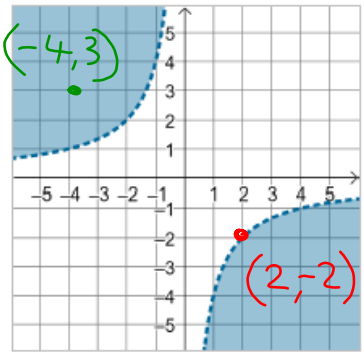


d)



Choose a point in the shaded region. Test to see if the statement is true. Dashed lines do NOT include the value, solid lines do include it.

10. Determine an algebraic representation of the graph on the right.



Find a "nice" point on the boundary of the region.

Example (2,-2)

Multiply the x and y to get the product

$(2)(-2) = -4$

Choose another point in the shaded region and compare it to the product.

Example $(-4,3) \Rightarrow (-4) \times (3) = -12$

which is less than the product

$\Rightarrow xy < -4$ — not equal to as the line is not solid.

11. By dividing both sides of the equation by x, we can express the relation $xy = 1$ as $y = \frac{1}{x}$. Using this representation of the relation,

a) explain why we cannot use an x-value of 0.

b) explain why the graph of the relation will not have any points with a y-value of 0.

c) explain what happens to the value of y as x approaches infinity.

d) explain what happens to the value of y as x approaches negative infinity.

e) explain what happens to the value of y as x approaches 0.

a) $x \neq 0$ because $\frac{1}{0}$ is undefined. It won't create a y-value, so it won't create a point on the graph.

b) No value x when put into $\frac{1}{x}$ can create a value of zero.

c) As $x \to \infty$, y remains positive, but approaches zero.

d) As $x \to -\infty$, y remains negative, but approaches zero.

e) As $x \to 0$, y approaches $-\infty$ if x is negative, y approaches $+\infty$ if x is positive.