Probability Distributions for Discrete Variables Extra Practice

MHR Page 190 #s 1 - 16

Solutions

1. An 8-sided die has its faces numbered 2, 4, 6 … 16. What is the expected outcome on a typical roll?

A 7 **B** 8

C 9 **D** 16

C

Each outcome is equally likely, so this is a uniform distribution.

$$E(X) = \frac{1}{n}\sum_{i=1}^{n} x_i$$

$$= \frac{1}{8}(2 + 4 + 6 + 8 + 10 + 12 + 14 + 16)$$

$$= \frac{1}{8}(72)$$

$$= 9$$

Yes, I know this seems strange as you can't get a 9 on this die!

2. The binomial and hypergeometric distributions are similar in that

A they both use independent trials

B they both use dependent trials

C they use the same formula for calculating the expectation

D they both involve counting successes

D

A - Binomial: independent trials

B - Hypergeometric: dependent trials

C - Different formulas

3. The expectation for a uniform distribution is calculated using

A $\frac{1}{n}\sum_{i=1}^{n} x_i$

B $\frac{ra}{n}$

C np

D $\frac{x}{n}$

A

B - Expected number for a hypergeometric distribution

C - Expected number for a binomial distribution

D - Nothing...

4. Counting the number of tails when a coin is flipped 20 times is an example of a

A binomial distribution

B hypergeometric distribution

C uniform distribution

D none of the above

A

This is an example of a binomial distribution because the trials are independent and the result of each trial is either success (head) or failure (not head).

5. The probability that exactly two students will be selected when five people are selected from four students and three teachers is

A $\frac{2}{5}$

B $\frac{{}_4C_2 \times {}_3C_3}{{}_7C_5}$

C ${}_5C_2\left(\frac{4}{7}\right)^2\left(\frac{3}{7}\right)^3$

D $\frac{5 \times 4}{7}$

B

Choosing 5 people from 7, so the denominator is ${}_7C_5$.

We want 2 students from 4 (${}_4C_2$) **AND** we want 3 teachers from 3 (${}_3C_3$).

6. A particular traffic light is programmed to be red 40% of the time. On his daily Monday to Friday commute to and from work, what is the expected number of times Jack can expect to have a red light?

This is a binomial distribution (independent events with success or failure).

$p = 0.4$, $n = 10$ (Mon-Fri commute there and back)

$E = np$

$E = 10(0.4)$

$E = 4$

Jack would expect to have 4 red lights.

7. Three cards are selected, without replacement, from the honour cards (10, J, Q, K, A) in a standard deck. What is the probability that two of them are face cards?

There are 20 honour cards (5 from each of the 4 suits) and 12 face cards (3 from each of the 4 suits) in a standard deck.

Choosing 3 cards from the total of 20 gives a denominator of ${}_{20}C_3$.

We want 2 face cards from the 12 (${}_{12}C_2$) **AND** we also want 1 non-face card from the 8 non-face cards (${}_{8}C_1$).

$$P(\text{2 honour cards}) = \frac{{}_{12}C_2 \times {}_{8}C_1}{{}_{20}C_3}$$

$$= \frac{66 \times 8}{1140}$$

$$= 0.463157894...$$

The probability that there are two face cards is about 0.4632

8. a) Is the situation in #7 modelled by a binomial or a hypergeometric distribution? Explain.

b) Describe how to change the situation to the binomial or hypergeometric distribution, as appropriate.

7. Three cards are selected, without replacement, from the honour cards (10, J, Q, K, A) in a standard deck. What is the probability that two of them are face cards?

a) It is modelled by a hypergeometric distribution because the events (selecting the cards) are dependent (upon what has already been chosen).

b) To make it binomial, you would replace each card after it is selected.

9. The beaver population in a particular provincial park is known to be 452. Two hundred beavers were caught and tagged. If 65 beavers were later caught and checked for tags, how many would you expect to be tagged?

This is an example of a hypergeometric distribution. As each beaver is caught, this will change the probabilities for future beavers that are caught of being tagged.

r = 65, a = 200, n = 452

$$E(X) = \frac{ra}{n}$$

$$= \frac{65(200)}{452}$$

$$= 28.76$$

I would expect 29 of the 65 caught beavers to be tagged.

10. Two dice are rolled a total of eight times, and the sum is recorded each time.

a) Show the probability distribution for a sum of 7.

b) Make a probability histogram of the distribution.

c) Determine and interpret the expected outcome.

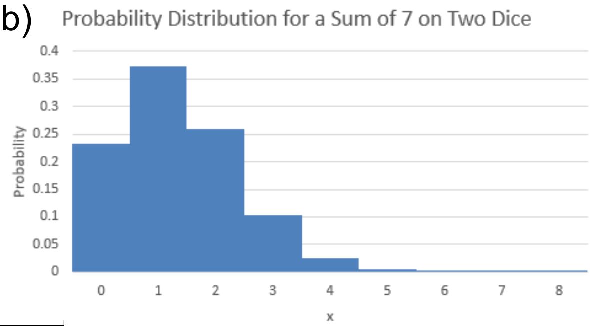
This is an example of a binomial distribution. Success = sum of 7, Failure = not sum of 7.

P(7) = 1/6 P(Not 7) = 5/6

a)

x	p	q	nCx	P(x)
0	0.167	0.833	1	0.232568
1	0.167	0.833	8	0.372109
2	0.167	0.833	28	0.260476
3	0.167	0.833	56	0.10419
4	0.167	0.833	70	0.026048
5	0.167	0.833	56	0.004168
6	0.167	0.833	28	0.000417
7	0.167	0.833	8	2.38E-05
8	0.167	0.833	1	5.95E-07

b)



c)

x	p	q	nCx	P(x)	x . P(x)
0	0.167	0.833	1	0.232568	0.000000
1	0.167	0.833	8	0.372109	0.372109
2	0.167	0.833	28	0.260476	0.520952
3	0.167	0.833	56	0.10419	0.312571
4	0.167	0.833	70	0.026048	0.104190
5	0.167	0.833	56	0.004168	0.020838
6	0.167	0.833	28	0.000417	0.002501
7	0.167	0.833	8	2.38E-05	0.000167
8	0.167	0.833	1	5.95E-07	0.000005

The expected outcome is the total of the x.P(x) column...

= 1.3333 $E(x) = np$

We would expect 1.33333 sums of 7 from 8 rolls of the two dice. $= 8(\frac{1}{6})$

11. A certain cell phone provider's help line is busy 95% of the time.

a) In 15 calls to the help line, what is the probability that it will be busy every time? at least 12 times?

b) What is the expected number of times a caller should expect the line to be busy in 15 attempts?

Binomial distribution as these are independent events with either a success or failure.

$p = 0.95$, $q = 0.05$, $n = 15$

a) $P(x \geq 12) = P(12) + P(13) + P(14) + P(15)$

$= {}_{15}C_{12}(0.95)^{12}(0.05)^3 + {}_{15}C_{13}(0.95)^{13}(0.05)^2 + {}_{15}C_{14}(0.95)^{14}(0.05)^1 + {}_{15}C_{15}(0.95)^{15}(0.05)^0$

$= 0.0307 + 0.1348 + 0.3658 + 0.4633$

$= 0.9946$

The probability that the line is busy at least 12 times is about 0.9946.

b) $E = np$

$E = 15(0.95)$

$E = 14.25$

The caller should expect the line to be busy 14.25 times.

12. Ten males and five females applied for four job promotions. The union's affirmative action committee is concerned that no females were hired, saying that at least one should have been female. Use appropriate calculations to support or refute their claim.

$$P(0 \text{ women}) = \frac{{}_5C_0 \times {}_{10}C_4}{{}_{15}C_4} = \frac{1 \times 210}{1365} = 0.1538$$

$$P(\text{At least 1 woman}) = 1 - \frac{{}_5C_0 \times {}_{10}C_4}{{}_{15}C_4} = 1 - 0.1538 = 0.8462$$

It is 5.5 times more likely that at least one woman would be promoted compared to no women being promoted, so the committee are correct in their supposition.

13. The incidence of a disease in the population is 12%. Six people are in an elevator.

a) What is the probability that at least two of them will have the disease?

b) What is the expected number of these people with the disease?

a) Use the indirect method $P(x \geq 2) = 1 - P(0) - P(1)$

$$P(x \geq 2) = 1 - {}_6C_0(0.12)^0(0.88)^6 - {}_6C_1(0.12)^1(0.88)^5$$

$$= 1 - 0.4644 - 0.3800$$

$$= 0.1556$$

The probability of at least two people having the disease is about 0.1556.

b) $E = np$

$E = 6(0.12)$

$E = 0.72$

The expected number of people with the disease is 0.72.

14. Eighteen of thirty players selected in the NHL first–round draft were Canadian. If seven drafted players are randomly selected, what is the probability that

a) only one is Canadian?

b) all are Canadian?

c) most of them are Canadian?

a) $$P(1\text{ Canadian}) = \frac{{}_{18}C_1 \times {}_{12}C_6}{{}_{30}C_7}$$

$$= 0.008169...$$

The probability that only one is Canadian is about 0.0082.

b) $$P(7\text{ Canadian}) = \frac{{}_{18}C_7 \times {}_{12}C_0}{{}_{30}C_7}$$

$$= 0.01563...$$

The probability that all seven are Canadian is about 0.01563.

c) Most are Canadian means, in this instance, four or more.

$$P(x \geq 4) = P(4) + P(5) + P(6) + P(7)$$

$$= \frac{{}_{18}C_4 \times {}_{12}C_3}{{}_{30}C_7} + \frac{{}_{18}C_5 \times {}_{12}C_2}{{}_{30}C_7} + \frac{{}_{18}C_6 \times {}_{12}C_1}{{}_{30}C_7} + \frac{{}_{18}C_7 \times {}_{12}C_0}{{}_{30}C_7}$$

$$= 0.3307 + 0.2778 + 0.1094 + 0.0156$$

$$= 0.7335$$

The probability that most are Canadian is about 0.7335.

15. If $n \div r > 200$, the binomial distribution can be used to approximate the hypergeometric distribution. Why would this be?

The binomial distribution can be used to approximate a hypergeometric distribution when the ratio of n ÷ r is large.

This is because when r is very small compared to n, the non-replacement of successes has little effect on the ratio of successes to the population.

In this case as the ratio is > 200, we can use the binomial distribution to represent a hypergeometric distribution.

To be a positive number we need to have either 4 positive numbers OR 2 positive and 2 negative numbers OR 4 negative numbers.

a) No repetition gives a Hypergeometric Distribution

$$= \frac{{}_6C_4 \times {}_8C_0}{{}_{14}C_4} + \frac{{}_6C_2 \times {}_8C_2}{{}_{14}C_4} + \frac{{}_6C_0 \times {}_8C_4}{{}_{14}C_4}$$

$$\approx 0.0150 + 0.4196 + 0.0700$$

$$\approx 0.5046$$

b) Repetition allowed gives a Binomial Distribution

$$= {}_4C_4\left(\frac{6}{14}\right)^4\left(\frac{8}{14}\right)^0 + {}_4C_2\left(\frac{6}{14}\right)^2\left(\frac{8}{14}\right)^2$$

$$+ {}_4C_0\left(\frac{6}{14}\right)^0\left(\frac{8}{14}\right)^4$$

$$\approx 0.1006 + 0.3599 + 0.0337$$

$$\approx 0.5002$$