Solutions

1. Classify each random variable as discrete or continuous.
   a) length of time you play in a hockey game
   b) number of times you successfully shoot a basket in a basketball game
   c) number of candies in a bag
   d) mass of candies in a bag

Count discrete data

Measure continuous data

a) Continuous

b) Discrete

c) Discrete

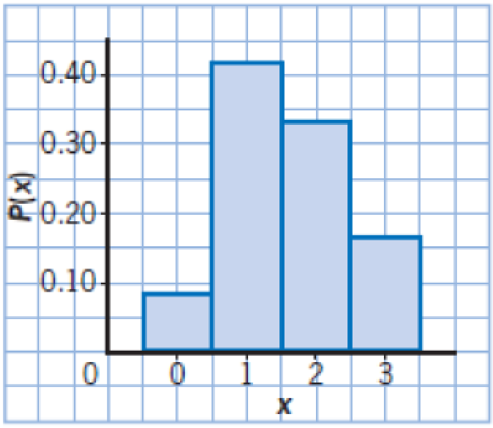
d) Continuous

2. Graph each distribution using a probability histogram.

a)

x	P(x)
0	$\frac{1}{12}$
1	$\frac{5}{12}$
2	$\frac{1}{3}$
3	$\frac{1}{6}$

Convert fractions to decimals to make graphing easier



3. Calculate the expectation for each distribution in #2.

E(x) = 0(1/12) + 1(5/12) + 2(1/3) + 3(1/6)

= 0 + 5/12 + 2/3 + 3/6

= 5/12 + 8/12 + 6/12

= 19/12

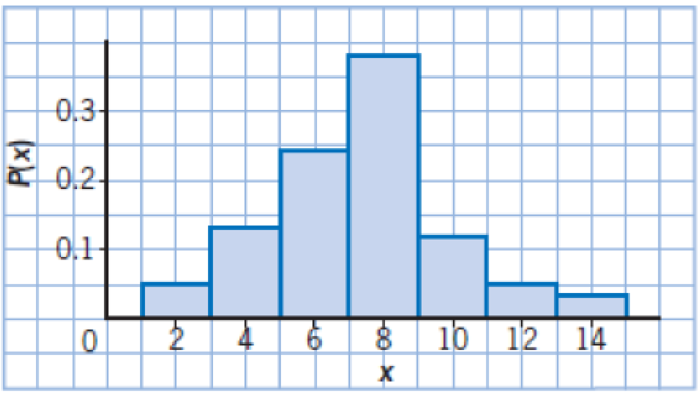
= 1.583...

The expected value is 1.58

2. Graph each distribution using a probability histogram.

b)

x	P(x)
2	0.05
4	0.13
6	0.24
8	0.38
10	0.12
12	0.05
14	0.03



3. Calculate the expectation for each distribution in #2.

E(x) = 2(0.05) + 4(0.13) + 6(0.24) + 8(0.38) + 10(0.12) + 12(0..05) + 14(0.03)

= 7.32 The expected value is 7.32

4. Describe the criteria for a distribution to be uniform.

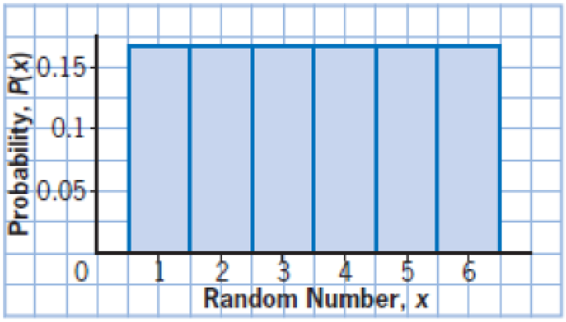
For a distribution to be uniform, the outcomes are all equally likely for a single trial.

5. A spinner has six equal sectors, numbered from 1 to 6.

a) Show the probability distribution for a single spin, using a table and a graph.

b) Calculate the expected outcome. Interpret its meaning.

Random Number, x	$P(x)$	$x \cdot P(x)$
1	$\frac{1}{6}$	$\frac{1}{6}$
2	$\frac{1}{6}$	$\frac{2}{6}$
3	$\frac{1}{6}$	$\frac{3}{6}$
4	$\frac{1}{6}$	$\frac{4}{6}$
5	$\frac{1}{6}$	$\frac{5}{6}$
6	$\frac{1}{6}$	$\frac{6}{6}$



$E(x) = \Sigma\, x.P(x)$

$= 1/6 + 2/6 + 3/6 + 4/6 + 5/6 + 6/6$

$= 21/6$

$= 3.5$ The expected outcome is 3.5

This means that the predicted average will be 3.5

6. An urn contains 25 balls, 40% of which are green. A contestant reaches in the urn to choose three balls; the contestant will win \$200 if he or she selects a green ball, but will lose \$120 for any other colour. Is each version of the game a fair game? Justify your response.

a) The ball is replaced after each draw.

b) The ball is not replaced after each draw.

a) Binomial: is replaced

Number of Green Balls	Amount (\$), x	$P(x)$	$x \cdot P(x)$
0	−360	$_3C_0(0.4)^0(0.6)^3$	−77.76
1	−40	$_3C_1(0.4)^1(0.6)^2$	−17.28
2	280	$_3C_2(0.4)^2(0.6)^1$	80.64
3	600	$_3C_3(0.4)^3(0.6)^0$	38.40
		Sum	24

b) Hypergeometric: not replaced

Number of Green Balls	Amount (\$), x	$P(x)$	$x \cdot P(x)$
0	−360	$_3C_0\left(\frac{15}{25}\right)\left(\frac{14}{24}\right)\left(\frac{13}{23}\right)$	−71.22
1	−40	$_3C_1\left(\frac{10}{25}\right)\left(\frac{15}{24}\right)\left(\frac{14}{23}\right)$	−18.26
2	280	$_3C_2\left(\frac{10}{25}\right)\left(\frac{9}{24}\right)\left(\frac{15}{23}\right)$	82.17
3	600	$_3C_3\left(\frac{10}{25}\right)\left(\frac{9}{24}\right)\left(\frac{8}{23}\right)$	31.30
		Sum	23.99

Calculations for the Amounts, x (\$)

0 Green = 0(200) + 3(-120), 1 Green = 200 + 2(-120)

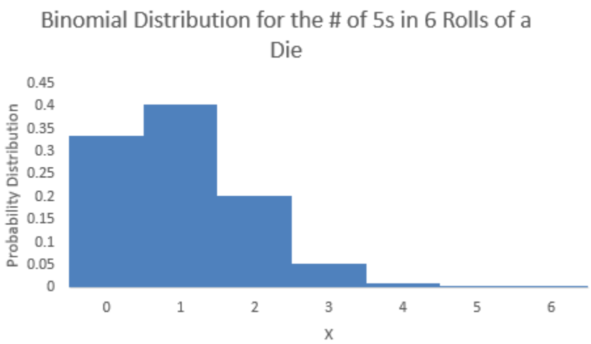
2 Green = 2(200) +1(-120), 3 Green = 3(200) + 0(-120)

Neither game is fair as the sums ≠ 0

7. Prepare a distribution table and probability histogram for the number of 5s when a die is rolled six times.

$$P(x) = {}_nC_x p^x q^{n-x}, \text{ where } n = 6,\ x = 0, 1, 2, 3, 4, 5, 6,\ p = \frac{1}{6}, \text{ and } q = \frac{5}{6}$$

x	p	q	nCx	P(x)
0	0.1667	0.8333	1	0.334898
1	0.1667	0.8333	6	0.401878
2	0.1667	0.8333	15	0.200939
3	0.1667	0.8333	20	0.053584
4	0.1667	0.8333	15	0.008038
5	0.1667	0.8333	6	0.000643
6	0.1667	0.8333	1	2.14E-05



8. The chart shows the percent of Canadians with each blood type.

Blood Type	Percent
O	46
A	42
B	9
AB	3

a) If 120 people are donating blood, what is the expected number of people with type O blood? Why would this be considered a binomial distribution?

b) Calculate the expected number of people with each of the other types of blood.

$E(x) = np$

a) $E(\text{Type O}) = 120(0.46)$

$= 55.2$ people

It is considered binomial because people's blood type is either O or not O.

b) $E(\text{Type A}) = 120(0.42)$

$= 50.4$ people

$E(\text{Type B}) = 120(0.09)$

$= 10.8$ people

$E(\text{Type AB}) = 120(0.03)$

$= 3.6$ people

9. A restaurant gives customers a card with each purchase; customers scratch a box to see if they have won a prize. Twelve percent of the cards are winners.

a) What is the probability of winning a prize only once in 10 tries?

b) What is the probability of winning a prize at least three times in 10 tries?

c) What is the expected number of winning cards in 10 tries?

a) $P(1) = {}_{10}C_1(0.12)^1(0.88)^9$

$= 10(0.12)(0.3164...)$

$= 0.379774...$ (0.3798)

b) Use the indirect method

$P(x \geq 3) = 1 - P(0) - P(1) - P(2)$

$= 1 - {}_{10}C_0(0.12)^0(0.88)^{10} - {}_{10}C_1(0.12)^1(0.88)^9 - {}_{10}C_2(0.12)^2(0.88)^8$

$= 0.108681...$ (0.1087)

c) $E(x) = np$

$= 10(0.12)$

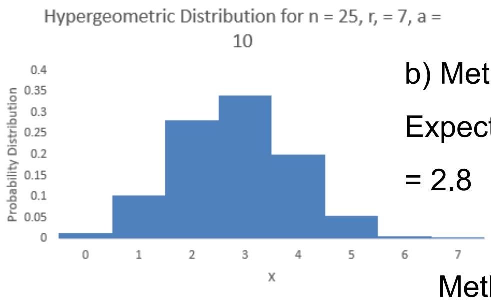
$= 1.2$

The expected number of winning tickets is 1.2

10. **a)** Prepare a table and a graph for a hypergeometric distribution with $n = 25$, $a = 10$, and $r = 7$.

b) Calculate the expected outcome using two methods.

x	n	r	a	n-a	r-x	aCx	n-aCr-x	nCr	P(x)	x . P(x)
0	25	7	10	15	7	1	6435	480700	0.013387	0
1	25	7	10	15	6	10	5005	480700	0.104119	0.104119
2	25	7	10	15	5	45	3003	480700	0.281121	0.562243
3	25	7	10	15	4	120	1365	480700	0.340753	1.022259
4	25	7	10	15	3	210	455	480700	0.198773	0.79509
5	25	7	10	15	2	252	105	480700	0.055045	0.275224
6	25	7	10	15	1	210	15	480700	0.006553	0.039318
7	25	7	10	15	0	120	1	480700	0.00025	0.001747



b) Method 1

Expected outcome is Σ x.P(x)

= 2.8

Method 2

E(x) = ra/n

= 7(10)/25

= 70 / 25

= 2.8

11. In a collection of 56 coins, 18 are rare. If you select 10 of the coins, what is the probability that

a) all of them are rare?

b) none of them is rare?

c) at least two of them are rare?

n= 56, r = 10, a = 18

a) $P(10) = \dfrac{{}_{18}C_{10} \times {}_{38}C_0}{{}_{56}C_{10}}$

$= 43758(1) / 3.5607... \times 10^{10}$

$= 1.2289 \times 10^{-6}$

b) $P(0) = \dfrac{{}_{18}C_0 \times {}_{38}C_{10}}{{}_{56}C_{10}}$

$= 1(472{,}733{,}756) / 3.5607... \times 10^{10}$

$= 0.0133$

c) $P(1) = \dfrac{{}_{18}C_1 \times {}_{38}C_9}{{}_{56}C_{10}}$

$= 18(163{,}011{,}640) / 3.5607... \times 10^{10}$

$= 0.0824$

Use the indirect method

P(at least 2 rare) = 1 - P(0) - P(1)

= 1 - 0.0133 - 0.0824

= 0.9043

12. The fisheries department caught and tagged 420 seals. Recently, 100 seals were caught and 42 had been tagged. Estimate the size of the seal population.

$n = ?$, $r = 420$, $a = 100$, $E(x) = 42$

$E(x) = ra/n$

$42 = 420(100)/n$

$42n = 42000$

$n = 42000/42$

$n = 1000$ seals

13. Classify each situation as uniform, binomial, hypergeometric, or none of these.

a) Forty-five percent of women aged 18 to 25 are currently enrolled in post-secondary education. The random variable is the number of women between the ages of 18 and 25, out of 25 polled, who attend post-secondary education.

Binomial

b) Twenty out of 30 people at a party are non-smokers. The random variable is the number of smokers in a selection of 8 partiers.

Hypergeometric

c) The flaws in pieces of timber average 0.2 per metre. The random variable is the number of flaws in the next 50 m of timber.

None

d) A spinner has 20 equally likely spaces, numbered from 1 to 20. The random variable is the number on which the spinner lands.

Uniform

14. a) Seven cards are dealt from a standard deck. What is the probability that five are face cards?

b) Seven cards are chosen from a standard deck, with replacement. What is the probability that five are face cards?

c) Compare the two answers in parts a) and b). Explain any differences.

a) P(5 Face Cards) = $\frac{{}_{12}C_5 \times {}_{40}C_2}{{}_{52}C_7}$

= 792(780)/133,784,560

= 0.004617... (0.0046)

b) P(5 Face Cards) = ${}_7C_5(3/13)^5(10/13)^2$

= $21(6.544... \times 10^{-4})(0.5917...)$

= 0.008132... (0.0081)

c) Part (a) does not have replacement so that is why the answer is lower. The probabilities change based upon what has previously been selected. Part (b) has replacement, the probabilities do not change.