Comparing and Selecting Discrete Probability Distributions

Lesson objectives

- I can compare the probability distributions of discrete random variables
- I can solve problems involving uniform, binomial, and hypergeometric distributions

1.1

Lesson objectives | Teachers' notes | Lesson notes

MHR Page 185 #s 1 - 8

Example

Compare Two Similar Distributions

a) Compare and contrast the following probability distributions. Include the values of the parameters.
- cutting five cards from a standard deck, with replacement, and counting the number of face cards
- dealing five cards at the same time from a standard deck and counting the number of face cards

b) Graph the two probability histograms.

c) How are the graphs alike? How are they different?

n, p, q

$P(\text{Face}) = \frac{12}{52} = \frac{3}{13}$

$P(\text{Not Face}) = \frac{40}{52} = \frac{10}{13}$

$P(0) = {}_5C_0 \left(\frac{3}{13}\right)^0 \left(\frac{10}{13}\right)^5$

$P(1) = {}_5C_1 \left(\frac{3}{13}\right)^1 \left(\frac{10}{13}\right)^4$

$P(2) = {}_5C_2 \left(\frac{3}{13}\right)^2 \left(\frac{10}{13}\right)^3$

$P(3) = {}_5C_3 \left(\frac{3}{13}\right)^3 \left(\frac{10}{13}\right)^2$

$P(4) = {}_5C_4 \left(\frac{3}{13}\right)^4 \left(\frac{10}{13}\right)^1$

$P(5) = {}_5C_5 \left(\frac{3}{13}\right)^5 \left(\frac{10}{13}\right)^0$

Your Turn

a) Use a Venn diagram to compare and contrast the probability distributions if a hat contains five male and six female names.
- Selecting four names with replacement, and counting the number of female names.
- Selecting four names without replacement, and counting the number of female names.

b) Graph the two probability histograms.

c) How are the graphs alike? How are they different?

$n = 52$

$a = 12$

$$P(0) = \frac{{}_{12}C_0 \times {}_{40}C_5}{{}_{52}C_5}$$

$$P(1) = \frac{{}_{12}C_1 \times {}_{40}C_4}{{}_{52}C_5}$$

$$P(2) = \frac{{}_{12}C_2 \times {}_{40}C_3}{{}_{52}C_5}$$

$$P(3) = \frac{{}_{12}C_3 \times {}_{40}C_2}{{}_{52}C_5}$$

$$P(4) = \frac{{}_{12}C_4 \times {}_{40}C_1}{{}_{52}C_5}$$

$$P(5) = \frac{{}_{12}C_5 \times {}_{40}C_0}{{}_{52}C_5}$$

Example

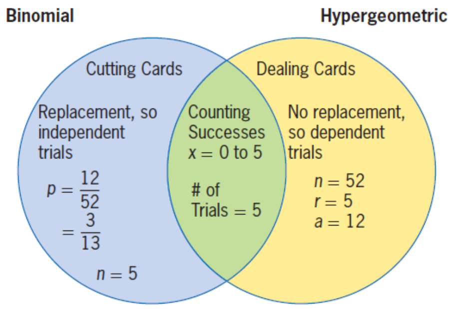
Compare Two Similar Distributions

a) Compare and contrast the following probability distributions. Include the values of the parameters.
- cutting five cards from a standard deck, with replacement, and counting the number of face cards
- dealing five cards at the same time from a standard deck and counting the number of face cards

b) Graph the two probability histograms.

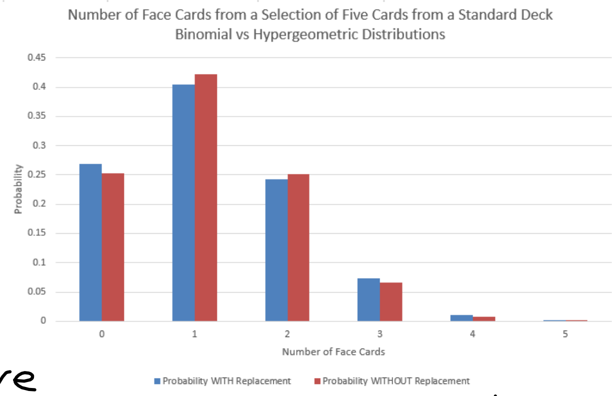
c) How are the graphs alike? How are they different?

a)



b)

Number of Face Cards	Probability WITH Replacement	Probability WITHOUT Replacement
0	0.269329074	0.253181273
1	0.403993612	0.421968788
2	0.242396167	0.25090036
3	0.07271885	0.066026411
4	0.010907828	0.007618432
5	0.00065447	0.000304737



c) The graphs both have a bell shape to them. The hypergeometric bars are taller than the binomial bars for $x = 1$ and $x = 2$. They are shorter for $x = 0, 3, 4,$ and 5. This occurs because when events are dependent there are fewer choices available, causing probabilities to increase.

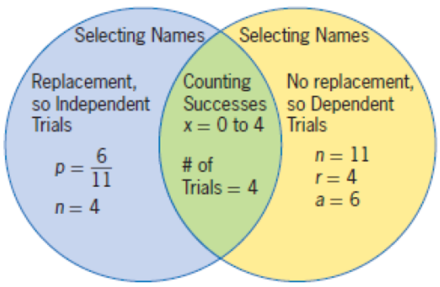
Your Turn

a) Use a Venn diagram to compare and contrast the probability distributions if a hat contains five male and six female names.
- Selecting four names with replacement, and counting the number of female names.
- Selecting four names without replacement, and counting the number of female names.

b) Graph the two probability histograms.

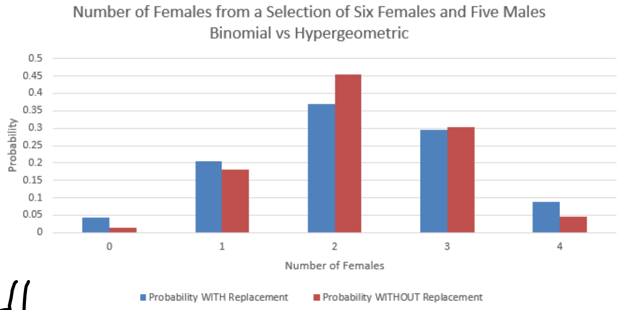
c) How are the graphs alike? How are they different?

a) **Binomial** **Hypergeometric**



b)

Number of Females	Probability WITH Replacement	Probability WITHOUT Replacement
0	0.042688341	0.015151515
1	0.204904037	0.181818182
2	0.368827266	0.454545455
3	0.295061813	0.303030303
4	0.088518544	0.045454545



c) The graphs both a bell shape to them. The hypergeometric bars are taller than the binomial bars for $x = 2$ and 3. They are shorter for $x = 0, 1,$ and 4. This is due to dependent events having an increase in probability when there are fewer choices.

Key Concepts

- The chart summarizes the general conditions of the distributions.

	Uniform	Binomial	Hypergeometric
Parameters and What They Represent	n = number of items	n = number of trials p = probability of success on an individual trial q = probability of failure on an individual trial	n = size of the population r = number of trials a = number of successful items available
Definition of Random Variable, x	Value of the outcome	Number of successful outcomes	Number of successful outcomes
Range of Values for x	Depends on the situation	$x = 0, 1, 2, \dots, n$	$x = 0, 1, 2, \dots, r$
Probability Formula	$P(x) = \frac{1}{n}$	$P(x) = {}_nC_x p^x q^{n-x}$	$P(x) = \frac{{}_aC_x \cdot {}_{n-a}C_{r-x}}{{}_nC_r}$
Expectation Formula	$E(X) = \frac{1}{n}\sum_{i=1}^{n} x_i$	$E(X) = np$	$E(X) = \frac{ra}{n}$
Identifying Characteristics	All items are equally likely A single trial	Trials are independent Successes are counted	Trials are dependent Successes are counted

R1. Refer to the graphic organizer in the Investigation on page 180, and to the general conditions chart in the Key Concepts above. Make a Venn diagram or a graphic organizer to compare and contrast the general conditions for the binomial and hypergeometric probability distributions.

Compare and Contrast

Concept 1	Concept 2
Binomial Distribution	Hypergeometric Distribution

How are they alike?

- Random variable, x, is the number of successes.
- Fixed number of trials.
- Successes are counted.
- Distribution is discrete.

How are they different?

Binomial Distribution	*Criteria*	Hypergeometric Distribution
Number of trials	• population	Size of population
Independent trials	• discrete or continuous	Dependent trials
$x = 0, 1, \dots, n$	• independence of trials	$x = 0, 1, \dots, r$
Probability of success does not change	• counting outcomes	Probability of success changes in every trial
$E(X) = np$	• number of trials	$E(X) = \frac{ra}{n}$
Parmeters: n, p, q	• what needs to be known?	Parmeters: n, r, a
	• expectation	
	• parameters (n, r)	

R2. Sam wrote that the difference between binomial and hypergeometric distributions is that with the binomial distribution each trial has the same probability, but with hypergeometric the individual probabilities change with the sampling. Is this an accurate statement? Explain.

No, not quite. For a binomial distribution, the probability of each **SUCCESS** is the same, but with hypergeometric the probability of success changes with each trial.