Solutions

1. A bag contains five red and six blue blocks. What is the probability of getting three red blocks if four blocks are randomly selected?

A $\frac{{}_{11}C_3 \times {}_{6}C_4}{{}_{18}C_4}$

B $\frac{{}_{5}C_3}{{}_{11}C_4}$

C $\frac{{}_{5}C_3}{{}_{6}C_4}$

D $\frac{{}_{5}C_3 \times {}_{6}C_1}{{}_{11}C_4}$ ← D

Numerator: Want 3 reds from 5 (${}_{5}C_3$) AND so must also have 1 blue from 6 (${}_{6}C_1$)

Denominator: Choosing a total of 4 blocks from 11 (${}_{11}C_4$)

2. Which is an example of a hypergeometric distribution?

A probability of the number of aces in a seven-card hand

B probability of each sum when two dice are rolled ← Binomial

C probability of a number being randomly chosen from the numbers 1 to 10 ← Uniform

D probability of the number of times a 3 occurs when rolling a die six times ← Binomial

A - Look for an example that has dependent events that have outcomes that are either success or failure.

3. Each expression represents the probability of a hypergeometric probability. State the values of the unknowns.

a) $\frac{{}_6C_3 \times {}_9C_2}{{}_nC_r}$

n = total number being chosen from...

= 6 + 9 = 15

r = total number being chosen...

= 3 + 2 = 5

b) $\frac{{}_aC_5 \times {}_7C_b}{{}_{10}C_6}$

a = number to choose from in the first batch...

= 10 - 7 = 3 ← Not actually a real answer as we can't have ${}_3C_5$

b = number chosen from the second batch...

= 6 - 5 = 1

c) $\frac{{}_6C_3 \times {}_cC_2}{{}_{25}C_d}$

c = number to choose from in the second batch...

= 25 - 6 = 19

d = total number being chosen...

= 3 + 2 = 5

4. Show the hypergeometric probability distribution for an experiment with

a) $n = 15, r = 4, a = 7.$

$$P(x) = \frac{{}_{a}C_{x} \cdot {}_{n-a}C_{r-x}}{{}_{n}C_{r}}$$

x	n	r	a	n-a	r-x	aCx	n-aCr-x	nCr	P(x)
0	15	4	7	8	4	1	70	1365	0.051282
1	15	4	7	8	3	7	56	1365	0.287179
2	15	4	7	8	2	21	28	1365	0.430769
3	15	4	7	8	1	35	8	1365	0.205128
4	15	4	7	8	0	35	1	1365	0.025641

Eg. $P(3) = \frac{{}_{7}C_{3} \times {}_{8}C_{1}}{{}_{15}C_{4}} = 0.2051282051\ldots$

4. Show the hypergeometric probability distribution for an experiment with

b) $n = 8, r = 4, a = 4.$

$$P(x) = \frac{{}_{a}C_{x} \cdot {}_{n-a}C_{r-x}}{{}_{n}C_{r}}$$

x	n	r	a	n-a	r-x	aCx	n-aCr-x	nCr	P(x)
0	8	4	4	4	4	1	1	70	0.014286
1	8	4	4	4	3	4	4	70	0.228571
2	8	4	4	4	2	6	6	70	0.514286
3	8	4	4	4	1	4	4	70	0.228571
4	8	4	4	4	0	1	1	70	0.014286

Eg. $P(3) = \frac{{}_{4}C_{3} \times {}_{4}C_{1}}{{}_{8}C_{4}} = 0.228571428 6\ldots$

5. A five-card hand is dealt from the honour cards in a standard deck (10, J, Q, K, A).

a) Show the probability distribution for the number of hearts in the hand.

n = 20, r = 5, a = 5

x	n	r	a	n-a	r-x	aCx	n-aCr-x	nCr	P(x)
0	20	5	5	15	5	1	3003	15504	0.193692
1	20	5	5	15	4	5	1365	15504	0.440209
2	20	5	5	15	3	10	455	15504	0.293473
3	20	5	5	15	2	10	105	15504	0.067724
4	20	5	5	15	1	5	15	15504	0.004837
5	20	5	5	15	0	1	1	15504	6.45E-05

Eg $P(2) = \frac{{}_5C_2 \times {}_{15}C_3}{{}_{20}C_5} = 0.2934726522\ldots$

5. A five-card hand is dealt from the honour cards in a standard deck (10, J, Q, K, A).

b) Calculate the expectation in two ways.

n = 20, r = 5, a = 5

Expectation $E(X) = \frac{ra}{n}$

E(x) = 5(5)/20

= 25/20

= 1.25

x	P(x)	x . P(x)
0	0.193692	0
1	0.440209	0.440209
2	0.293473	0.586945
3	0.067724	0.203173
4	0.004837	0.01935
5	6.45E-05	0.000322

Sum of x.P(x) = 1.25

6. In a box of 20 light bulbs, five are defective. Three light bulbs are selected at random.

a) What is the probability that at least one is defective?

b) What is the expected number of defective light bulbs?

c) What is the meaning of $P(3)$ in this context?

$n = 20$, $r = 3$, $a = 5$

Use the indirect method. Find P(0) and then subtract from 1 (total of all probabilities)

a)

x	n	r	a	n-a	r-x	aCx	n-aCr-x	nCr	P(x)
0	20	3	5	15	3	1	455	1140	0.399123

P(at least one defective) = 1 - P(0)

= 1 - 0.399123

= 0.600877 (0.6009)

b) E(x) = ra/n

= 3(5)/20

= 15/20

= 0.75

c) P(3) means the probability of choosing 3 defective lightbulbs.

7. In the card game of bridge, 13 cards are dealt to each player. Find the probability of each of the following hands:

a) 4 aces

All 4 Aces → ${}_4C_4$

9 other cards → ${}_{48}C_9$

All the possible 13 card hands → ${}_{52}C_{13}$

$$P(4\text{ Aces}) = \frac{{}_4C_4 \times {}_{48}C_9}{{}_{52}C_{13}}$$

P(4 Aces) = (1)(1,677,106,640) / 6.35013... x 10^{11}

= 0.002641... (0.0026)

7. In the card game of bridge, 13 cards are dealt to each player. Find the probability of each of the following hands:

Use the indirect method

b) at least 1 king

0 Kings → $_4C_0$

13 other cards (anything but Kings) → $_{48}C_{13}$

All the possible 13 card hands → $_{52}C_{13}$

$$P(\text{no Kings}) = \frac{_4C_0 \times {_{48}C_{13}}}{_{52}C_{13}}$$

P(no Kings) = (1)(1.92928... x 10^{11}) / 6.35013... x 10^{11}
= 0.3038...

P(at least one King) = 1 - P(no Kings)
= 1 - 0.3038
= 0.6962

7. In the card game of bridge, 13 cards are dealt to each player. Find the probability of each of the following hands:

c) 5 clubs, 8 diamonds

5 Clubs → $_{13}C_5$

8 Diamonds → $_{13}C_8$

0 other cards → $_{26}C_0$

All the possible 13 card hands → $_{52}C_{13}$

$$P(\text{5 clubs, 8 diamonds}) = \frac{_{13}C_5 \times {_{13}C_8} \times {_{26}C_0}}{_{52}C_{13}}$$

P(5 Cs, 8 Ds) = (1287)(1287)(1) / 6.35013... x 10^{11}
= 0.000002608...(2.608 x 10^{-6})

9. In a provincial park, 200 foxes are tagged. In 100 sightings, 14 were tagged. Estimate the size of the fox population.

n = ?, r = 100, a = 200, E(x) = 14

a is the number originally tagged

r is our sample size

E(x) is our expected number of tagged in our sample

$E(x) = ra/n$

$14 = 100(200)/n$

$14n = 20000$

$n = 20000/14$

$n = 1428.571429...$

The fox population is about 1429

OR Proportion statement

$$\frac{14}{100} = \frac{200}{n}$$

Tagged / Sightings → $\frac{14}{100}$

Total tagged / Total sightings → $\frac{200}{n}$ ← Population

10. Application Wildlife officials tagged 80 deer in an area that had approximately 120 deer.

a) If they later took a sample of 25 deer, how many would they expect to have been tagged?

b) Should the officials be surprised if the sample has fewer than 13 tagged deer? Explain your thinking.

n = 120, r = 25, a = 80, E(x) = ?

a is the number originally tagged

r is our sample size

n is our population of deer

$E(x) = ra/n$

$= 25(80)/120$

$= 2000/120$

$= 16.6666...$

The expected number of deer is about 17

b) They should be a bit surprised as the expected number is 17 which is quite a bit higher than their sample.