Solutions

1. Which expression best represents the probability of three successes in seven independent trials in a binomial distribution?

A $_7C_3p^3q^4$ B $_7C_3p^4q^3$
C $_7C_4p^3q^4$ D $_7C_4p^4q^3$

A - $_nC_x(p)^x(1-p)^{n-x}$

Binomial means two outcomes: success or failure

7 choose 3 gives $_7C_3$: the number of ways of success

p^3 gives the probability of success (p) raised to the power of the number of successes (x)

q^4 gives the probability of failure (1-p) raised to the number of failures (n-x)

2. What is the expectation for a binomial distribution with $p = 0.5$ and $n = 8$?

A 0.4 B 4
C 16 D 0.0625

E(x) = np

= 8(0.5)

= 4 ⟶ B

3. Which of the following is an example of a binomial distribution?

A probabilities of the number of queens in a five-card hand
B probability of each sum when two dice are rolled
C probability of each lane for a 100 m race
D probabilities of the number of times a 5 occurs when spinning a spinner six times

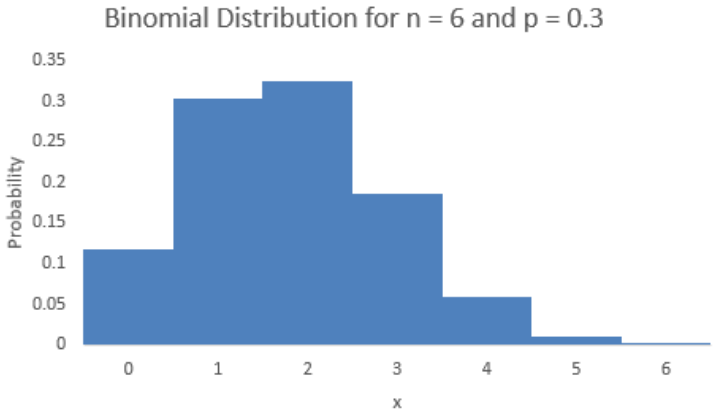
D - Looking for independent trials that will produce either a success or a failure

5. Prepare a probability table and a graph for a binomial distribution with

a) $n = 6$ and $p = 0.3$

$P(x) = {}_nC_x p^x q^{n-x}$, where $n = 6$, $x = 0, 1, 2, 3, 4, 5, 6$, $p = 0.3$, and $q = 0.7$.

x	p	q	nCx	P(x)
0	0.3	0.7	1	0.117649
1	0.3	0.7	6	0.302526
2	0.3	0.7	15	0.324135
3	0.3	0.7	20	0.18522
4	0.3	0.7	15	0.059535
5	0.3	0.7	6	0.010206
6	0.3	0.7	1	0.000729

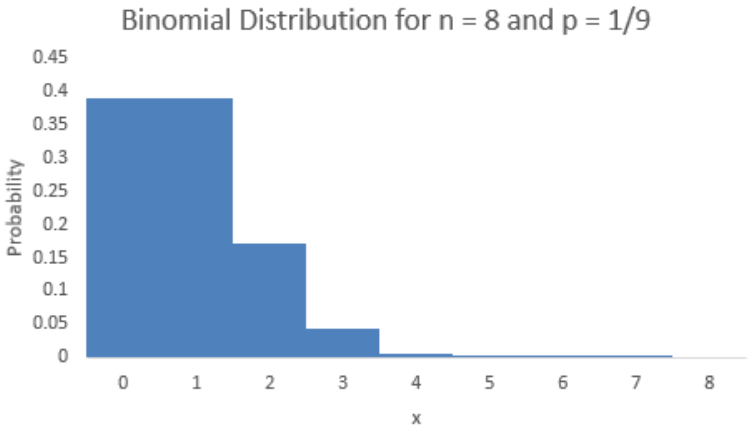


5. Prepare a probability table and a graph for a binomial distribution with

b) $n = 8$ and $p = \frac{1}{9}$

$P(x) = {}_nC_x p^x q^{n-x}$, where $n = 8$, $x = 0, 1, 2, 3, 4, 5, 6, 7, 8$, $p = \frac{1}{9}$, and $q = \frac{8}{9}$.

x	p	q	nCx	P(x)
0	0.111	0.889	1	0.389744
1	0.111	0.889	8	0.389744
2	0.111	0.889	28	0.170513
3	0.111	0.889	56	0.042628
4	0.111	0.889	70	0.006661
5	0.111	0.889	56	0.000666
6	0.111	0.889	28	4.16E-05
7	0.111	0.889	8	1.49E-06
8	0.111	0.889	1	2.32E-08



6. What is the expected number of times a 6 appears when rolling a die 2000 times?

n = 2000, p = 1/6

$E(x) = np$

$= 2000(1/6)$

$= 333.333...$

The expected number of times a 6 will appear in 2000 rolls is 333.333

7. In a family of five children, what is the probability that there are exactly
   a) two girls?
   b) three boys?

$$P(x) = {}_nC_x p^x q^{n-x}$$

n = 5, x = 2, p = 0.5, q = 0.5

a) $P(2) = {}_5C_2(0.5)^2(0.5)^3$

$= 10(0.25)(0.125)$

$= 0.3125$

n = 5, x = 3, p = 0.5, q = 0.5

b) $P(3) = {}_5C_3(0.5)^3(0.5)^2$

$= 10(0.125)(0.25)$

$= 0.3125$

8. Six people are asked to choose a number between 1 and 20. What is the probability that

a) two people choose the number 9?

$$P(x) = {}_nC_x p^x q^{n-x}$$

n = 6, x = 2, p = 0.05, q = 0.95

$P(2) = {}_6C_2(0.05)^2(0.95)^4$

$= 15(0.0025)(0.8145...)$

$= 0.03054...$

8. Six people are asked to choose a number between 1 and 20. What is the probability that

b) at least two people choose the number 9?

$$P(x) = {}_nC_x p^x q^{n-x}$$

Use the indirect method. Solve for x = 0 and x = 1 then subtract from 1 (the probability of all outcomes)

n = 6, x = 0, p = 0.05, q = 0.95

$P(0) = {}_6C_0(0.05)^0(0.95)^6$

$= 1(1)(0.73509...)$

$= 0.73509...$

n = 6, x = 1, p = 0.05, q = 0.95

$P(1) = {}_6C_1(0.05)^1(0.95)^5$

$= 6(0.05)(0.77378...)$

$= 0.23213...$

P(at least 2 choose 9) = 1 - P(0) - P(1)

= 1 - 0.73509 - 0.23213

= 0.03278

9. Two dice are rolled repeatedly and their sum is recorded.

a) Show the probability distribution for the number of sums of 7 in five rolls.

b) Graph the distribution with a probability histogram.

c) Verify the formula $E(X) = np$.

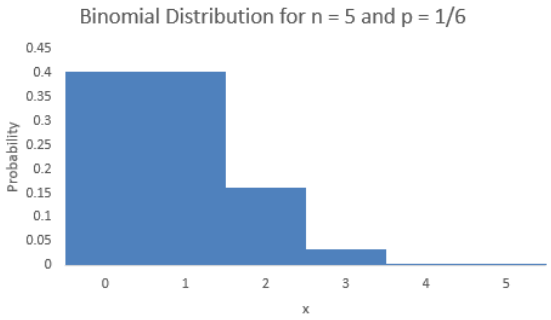
$$P(x) = {}_nC_x p^x q^{n-x}$$

n = 5, x = 0, 1, 2, 3, 4, 5, p = 1/6, q = 5/6

a)

x	p	q	nCx	P(x)
0	0.167	0.833	1	0.401878
1	0.167	0.833	5	0.401878
2	0.167	0.833	10	0.160751
3	0.167	0.833	10	0.03215
4	0.167	0.833	5	0.003215
5	0.167	0.833	1	0.000129

b)



c) E(x) = np

= 5(1/6)

= 0.833333

x	P(x)	x . P(x)
0	0.401878	0
1	0.401878	0.401878
2	0.160751	0.321502
3	0.03215	0.096451
4	0.003215	0.01286
5	0.000129	0.000643

Sum of x.P(x)

= 0.83333