# Solutions

1. Classify each of the random variables as discrete or continuous:
   a) the number of points scored in a basketball game
   b) the length of time players played in a basketball game
   c) the mass of the weights in a weight room
   d) the number of windows in the classrooms in a school
   e) the area of the windows in the classrooms in a school

a) discrete
b) continuous
c) discrete has a distinct mass, so is considered discrete, not continuous
d) discrete
e) continuous

Generally: discrete data is counted, continuous data is measured.

2. Which of the following is a false statement about expectation?

**A** The sum in the expected value calculations is equal to 1.

**B** $E(X) = \sum_{i=1}^{n} x_i \cdot P(x_i)$

**C** It is the predicted average of all possible outcomes.

**D** It is equal to the mean of the outcomes weighted according to their respective frequencies.

A is false

The sum of the expected values does not have to equal one. It is the predicted average, which does not have to equal one.

3. In Example 2 on page 148, what is the discrete random variable?

**A** $x$

**B** $P(x)$

**C** the number of girls in a family of three children

**D** the expected number of girls in a family of three children

### Example 2

**Expectation of a Probability Distribution**

a) Make a tree diagram and show the probability distribution for the number of girls in a family of three children.

b) Make a probability histogram for this distribution.

c) Calculate the weighted mean number of girls in a "typical" family of three children.

d) Calculate the expectation for the number of girls in a family of three children. Compare it to the weighted mean.

e) Interpret the results in parts c) and d).

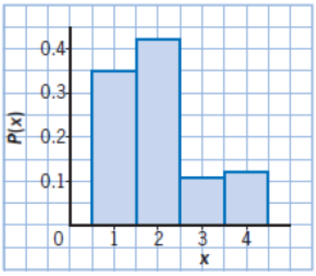
C - This is what can change (vary)

4. Draw a probability histogram for each of the distributions.

**Recall:**
Histograms have the bars touching

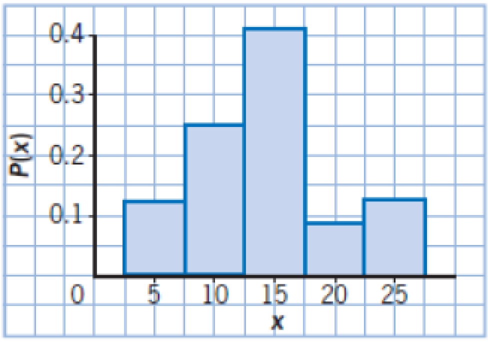
a)

| x | P(x) |
|---|---|
| 1 | 0.35 |
| 2 | 0.42 |
| 3 | 0.11 |
| 4 | 0.12 |




b)

| x | P(x) |
|---|---|
| 5 | $\frac{1}{8}$ |
| 10 | $\frac{1}{4}$ |
| 15 | $\frac{5}{12}$ |
| 20 | $\frac{1}{12}$ |
| 25 | $\frac{1}{8}$ |




It is easier to graph with decimals, so convert the fractions to decimals

5. Calculate the expectation for each of the distributions.

$$E(X) = \sum_{i=1}^{n} x_i \cdot P(x_i)$$

Multiply x by P(x) for each row and add them up

a)

| x | P(x) |
|---|---|
| 1 | 0.3 |
| 2 | 0.2 |
| 3 | 0.1 |
| 4 | 0.4 |

$$\sum_{x=1}^{4} x \cdot P(x) = 1(0.3) + 2(0.2) + 3(0.1) + 4(0.4)$$

$$= 0.3 + 0.4 + 0.3 + 1.6$$

$$= 2.6$$

The expected value is 2.6

5. Calculate the expectation for each of the distributions.

$$E(X) = \sum_{i=1}^{n} x_i \cdot P(x)_i$$

b)

| x | P(x) |
|---|---|
| 0 | $\frac{1}{5}$ |
| 2 | $\frac{3}{10}$ |
| 4 | $\frac{1}{5}$ |
| 6 | $\frac{1}{10}$ |
| 8 | $\frac{1}{10}$ |
| 10 | $\frac{1}{10}$ |

Multiply x by P(x) for each row and add them up

E(x) = 0(0.2) + 2(0.3) + 4(0.2) + 6(0.1) + 8(0.1) + 10(0.1)

= 0 + 0.6 + 0.8 + 0.6 + 0.8 + 1

= 3.8

The expected value is 3.8

6. **Communication** The distribution of marble sizes in a bag is shown in the table.

| Diameter (mm) | Frequency | Probability, $P(x)$ |
|---|---|---|
| 12.0 | 5 | $\frac{1}{12}$ |
| 13.0 | 11 | $\frac{11}{60}$ |
| 14.0 | 24 | $\frac{2}{5}$ |
| 20.0 | 15 | $\frac{1}{4}$ |
| 25.0 | 5 | $\frac{1}{12}$ |

Total Freq = 60

a) Identify the random variable.

b) Is the random variable discrete? Explain.

c) Draw a probability histogram for this distribution.

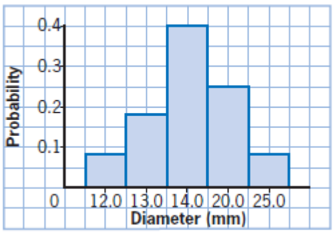
d) Describe what each bar in the histogram represents.

e) Calculate the weighted mean of the diameters. How does this relate to the expectation?

a) The diameter of the marble.

b) Yes. The diameter of the marble has already been measured.

c) To draw the histogram we need the probabilities (Probability = Freq/Total Freq)




d) Each bar's area represents the probability of each outcome. As the width of each bar is one, the probability is equal to the height of each bar.

e) = [12.0(5) + 13.0(11) + 14.0(24) + 20.0(15) + 25.0(5)]/60

= [60 + 143 + 336 + 300 + 125]/60

= 964/60

= 16.0666... The weighted mean is the same as the expected diameter

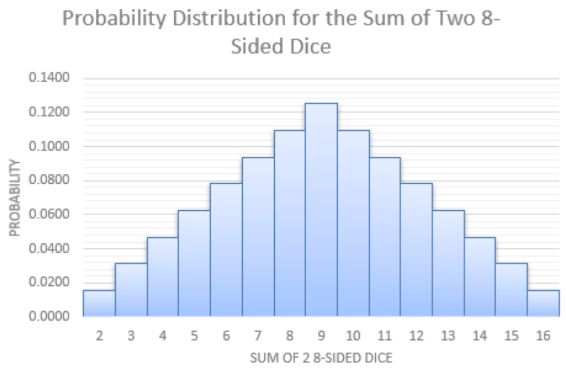
**7. Application** Two 8-sided dice are rolled.

a) Show the probability distribution for the sums of the two dice.

b) Draw a probability histogram by hand or using technology.

c) Calculate the expectation. Explain its meaning in this context.

| First Die / Second Die | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 |
| 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
| 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 |
| 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 |
| 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 |
| 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 |




E(x) = [2(1) + 3(2) + 4(3) + 5(4) + 6(5) +7(6) + 8(7) + 9(8) + 10(7) + 11(6) + 12(5) + 13(4) + 14(3) + 15(2) + 16 (1)]/64

= [2 + 6 + 12 + 20 + 30 + 42 + 56 + 72 + 70 + 66 + 60 + 52 + 42 + 30 + 16]/64

= 576/64

= 9

The expected sum of the two dice is 9

9. **Thinking** A school is holding a fundraising raffle. The first prize is \$500, the three second prizes are \$100 each, and the five third prizes are \$50 each. A total of 2000 tickets were sold at \$5 each.

a) What is the probability of winning a prize?

b) What is the expected payout per ticket?

c) What is the expected profit per ticket?

d) What price should have been charged to have a 90% profit per ticket?

a) 9 prizes for 2000 tickets... P(Prize) = 9/2000

b) E(x) = [1(500) + 3(100) + 5(50)]/2000

= [500 + 300 + 250]/2000

= 1050/2000

= \$0.525 The expected payout is \$0.525

c) Profit = Cost - Payout

= 5 - 0.525

= \$4.475

d) 90% would mean making 0.9(5) = \$4.50 per ticket.

Solving Cost - Payout = Profit

x - 0.525 = 4.5

x = 5.025 = \$5.03 would be the cost of the ticket