Prerequisite Skills

Lesson objectives

- I know how to calculate simple probability, with mutually exclusive events, and with combinations
- I know how to distinguish independent and dependent events
- I know how to evaluate expressions, use the binomial theorem, and how to graph a histogram

1.1

Lesson objectives | Teachers' notes | Lesson notes

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Solutions

1. When selecting a card from a standard deck, what is the probability that it is
   a) a king?
   b) a red card?
   c) a spade?

a) 4 Kings in a standard deck of 52 cards.

$$\frac{4}{52} = \frac{1}{13}$$

b) 26 red cards in a standard deck of 52 cards.

$$\frac{26}{52} = \frac{1}{2}$$

c) 13 Spades in a standard deck of 52 cards.

$$\frac{13}{52} = \frac{1}{4}$$

2. An experiment involves rolling three dice and recording the sum.
   a) Calculate the probability of each sum.
   b) Verify that the sum of the probabilities is 1.

Smallest total = 3 (1,1,1)

Largest total = 18 (6,6,6)

Sum	Possible Groupings	Number of Outcomes	Probability
3	(1,1,1)	1	$\frac{1}{216}$
4	(1,2,1)	3	$\frac{3}{216}$
5	(1,3,1), (1,2,2)	6	$\frac{6}{216}$
6	(1,4,1), (1,3,2), (2,2,2)	10	$\frac{10}{216}$
7	(1,4,2), (1,3,3), (5,1,1), (3,2,2)	15	$\frac{15}{216}$
8	(1,4,3), (1,2,5), (1,1,6), (4,2,2), (3,3,2)	21	$\frac{21}{216}$
9	(6,2,1), (5,3,1), (5,2,2), (4,4,1), (4,3,2), (3,3,3)	25	$\frac{25}{216}$
10	(6,3,1), (6,2,2), (5,3,2), (5,4,1), (4,4,2), (4,3,3)	27	$\frac{27}{216}$
11	(6,4,1), (6,3,2), (5,5,1), (5,4,2), (5,3,3), (4,4,3)	27	$\frac{27}{216}$
12	(6,5,1), (6,4,2), (6,3,3), (5,5,2), (5,4,3), (4,4,4)	25	$\frac{25}{216}$
13	(6,6,1), (6,5,2), (6,4,3), (5,5,3), (5,4,4)	21	$\frac{21}{216}$
14	(6,4,4), (6,5,3), (5,5,4), (6,6,2)	15	$\frac{15}{216}$
15	(6,6,3), (6,4,5), (5,5,5)	10	$\frac{10}{216}$
16	(6,6,4), (6,5,5)	6	$\frac{6}{216}$
17	(6,6,5)	3	$\frac{3}{216}$
18	(6,6,6)	1	$\frac{1}{216}$

When the numbers are all the same there is one way to arrange that group. When two numbers are the same there are 3!/2! = 3 ways to arrange that group. When all three numbers are different there are 3! ways to arrange that group.

b) Add up all the probabilities and they total 216/216.

3. In an experiment, a lab rat searches for food behind one of eight doors. Three doors are red, and the remaining doors are green.

a) What is the probability that the food was placed behind a red door?

b) What is the probability that the rat will select the correct door?

3 Red Doors

5 Green Doors (8 - 3)

1 Correct Door

a) $P(\text{Red}) = \frac{3}{8}$

b) $P(\text{Correct}) = \frac{1}{8}$

4. A bag contains six red, one green, four blue, and three yellow marbles. A marble is selected at random. What is the probability that the marble is

a) red or green?

b) neither red nor blue?

Total = 6 + 1 + 4 + 3 = 14 marbles

a) $P(\text{Red or Green}) = \frac{6}{14} + \frac{1}{14}$

$= \frac{7}{14}$

$= \frac{1}{2}$

The probability of selecting a red or green marble is 1/2.

b) $P(\text{Neither Red nor Blue}) = 1 - \frac{6}{14} - \frac{4}{14}$

$= 1 - \frac{10}{14}$

$= \frac{4}{14}$

$= \frac{2}{7}$

The probability of selecting neither a red nor a blue marble is 2/7.

5. If a deck containing only the face cards is shuffled and one card is selected, what is the probability that the card is

a) a queen or a king?

b) a red card or the queen of spades?

c) a red card and a spade?

Face cards are J,Q,K

4 Jacks, 4 Queens, 4 Kings = 12 in total

a) $P(Q \text{ or } K) = \frac{4}{12} + \frac{4}{12}$

$= \frac{8}{12}$

$= \frac{2}{3}$

b) $P(\text{Red or Q of S}) = \frac{6}{12} + \frac{1}{12}$

$= \frac{7}{12}$

c) P(Red and Spade) = 0

Spades are black, not red, so you can't select both as they are mutually exclusive.

6. Classify each pair of events as independent or dependent.

a) rolling two dice

b) selecting two cards at the same time from a standard deck

c) flipping a head on one coin and a tail on another

d) selecting two males from a list of four males and five females

a) Independent - Result on one die does not effect the other.

b) Dependent - The act of dealing the first card effects the probability of what the second card could be.

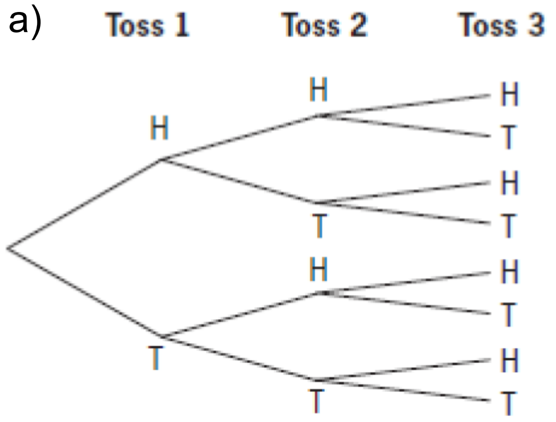
c) Independent - Result of the first coin does not effect the result on the second coin.

d) Dependent - The act of selecting the first person changes the probability of selecting the second person.

7. a) Use a tree diagram to illustrate the probabilities associated with the number of heads when three coins are flipped.

b) Are the events (head, tail, tail) independent or dependent? Explain.

a)



b) (H,T,T) is an independent event. The result of the first flip, does not effect the result of the second; and the result of the second flip does not effect the result of the third.

8. A card game uses only the hearts. Players select two cards without replacement. What is the probability that a player will

a) select a queen followed by a king?

b) select a queen and a king?

c) not select a face card on either draw?

13 Hearts in a standard deck.

Only one of each rank of card.

3 Face cards in total (J,Q,K).

a) P(Q then K) $= \frac{1}{13} \times \frac{1}{12}$

$= \frac{1}{156}$

b) P(Q and K) $= \frac{2}{13} \times \frac{1}{12}$

$= \frac{2}{156}$

$= \frac{1}{78}$

c) P(Not a Face) $= \frac{10}{13} \times \frac{9}{12}$

$= \frac{90}{156}$

$= \frac{15}{26}$

9. Twelve men and 10 women apply to attend a special event. Six names are selected.

a) In how many ways could three men and three women be selected?

b) In how many ways could more men than women be selected?

12 men, 10 women, 22 total.

a) ${}_{12}C_3 \times {}_{10}C_3$

$= 220 \times 120$

$= 26{,}400$

Three men and three women can be selected in 26,400 ways.

b) 4 men and 2 women **OR** 5 men and 1 woman **OR** 6 men and 0 women

$= {}_{12}C_4 \times {}_{10}C_2 + {}_{12}C_5 \times {}_{10}C_1 + {}_{12}C_6 \times {}_{10}C_0$

$= 495 \times 45 + 792 \times 10 + 924 \times 1$

$= 22275 + 7920 + 924$

$= 31{,}119$

More men than women can be selected in 31,119 different ways.

10. A cookie jar contains three chocolate chip, four peanut butter, and six butterscotch cookies. Hansa reaches in and grabs a handful of five cookies. In how many ways could she select

a) two chocolate chip, two peanut butter, and one butterscotch cookie?

b) no chocolate chip cookies?

c) at least one of each type of cookie?

13 cookies in the jar in total.

a) ${}_3C_2 \times {}_4C_2 \times {}_6C_1$

$= 3 \times 6 \times 6$

$= 108$ ways

b) If choosing no chocolate chip cookies then Hansa is selecting in ${}_{10}C_5 = 252$ ways.

c) To have at least one of each type we need to calculate each case.

1: 1 CC, 1 PB, 3 BS = ${}_3C_1 \times {}_4C_1 \times {}_6C_3 = 240$

2: 1 CC, 2 PB, 2BS = ${}_3C_1 \times {}_4C_2 \times {}_6C_2 = 270$

3: 1 CC, 3 PB, 1 BS = ${}_3C_1 \times {}_4C_3 \times {}_6C_1 = 72$

4: 2 CC, 1 PB, 2 BS = ${}_3C_2 \times {}_4C_1 \times {}_6C_2 = 180$

5: 2 CC, 2 PB, 1 BS = ${}_3C_2 \times {}_4C_2 \times {}_6C_1 = 108$

6: 3CC, 1 PB, 1 BS = ${}_3C_3 \times {}_4C_1 \times {}_6C_1 = 24$

Total $= 240 + 270 + 72 + 180 + 108 + 24$

$= 894$ ways.

11. In how many ways could 15 different books be divided equally
a) among five different people?
b) among three different people?

There is a total of 15! ways to arrange the books.

The order we give the books out is not important.

a) Giving equal groups to 5 people means 3 books each.

First person gets ${}_{15}C_3$ different ways, Second gets ${}_{12}C_3$, Third gets ${}_{9}C_3$, Fourth gets ${}_{6}C_3$, and Fifth gets ${}_{3}C_3$.

Total arrangements = ${}_{15}C_3 \times {}_{12}C_3 \times {}_{9}C_3 \times {}_{6}C_3 \times {}_{3}C_3$

= 455 x 220 x 84 x 20 x 1

= 168,168,000 ways.

b) Giving equal groups to 3 people means 3 books each.

First person gets ${}_{15}C_5$ different ways, Second gets ${}_{10}C_5$, and the Third gets ${}_{5}C_5$.

Total arrangements = ${}_{15}C_5 \times {}_{10}C_5 \times {}_{5}C_5$

= 3003 x 252 x 1

= 756,756 ways.

12. Evaluate.

a) ${}_{12}C_5 \times {}_{8}C_3$

b) $\frac{{}_{8}C_5}{{}_{12}C_7}$

c) $\frac{{}_{7}C_3 \times {}_{9}C_5}{{}_{16}C_8}$

d) ${}_{5}C_2(0.6)^2(0.4)^3$

e) ${}_{6}C_4\left(\frac{1}{3}\right)^4\left(\frac{2}{3}\right)^2$

a) = 792 x 56

= 44,352

b) = 56 ÷ 792

= 0.070707...

c) = 35 x 126 ÷ 12,870

= 0.342657342...

d) = 10 x 0.36 x 0.064

= 0.2304

e) = $15 \times \frac{1}{81} \times \frac{4}{9}$

= 0.0823...

13. Use the binomial theorem to expand.

a) $(x + y)^4$

Example:
$(a + b)^3 = {}_3C_0a^3 + {}_3C_1a^2b + {}_3C_2ab^2 + {}_3C_3b^3$
$= a^3 + 3a^2b + 3ab^2 + b^3$

a) $= {}_4C_0(x)^4(y)^0 + {}_4C_1(x)^3(y)^1 + {}_4C_2(x)^2(y)^2 + {}_4C_3(x)^1(y)^3 + {}_4C_4(x)^0(y)^4$

$= 1x^4y^0 + 4x^3y^1 + 6x^2y^2 + 4x^1y^3 + 1x^0y^4$

$= x^4 + 4x^3y + 6x^2y^2 + 4xy^3 + y^4$

b) $(4x + 3y)^5$

b) $= {}_5C_0(4x)^5(3y)^0 + {}_5C_1(4x)^4(3y)^1 + {}_5C_2(4x)^3(3y)^2 + {}_5C_3(4x)^2(3y)^3 + {}_5C_4(4x)^1(3y)^4 + {}_5C_5(4x)^0(3y)^5$

$= 1(1024x^5)(1y^0) + 5(256x^4)(3y^1) + 10(64x^3)(9y^2) + 10(16x^2)(27y^3) + 5(4x^1)(81y^4) + 1(1x^0)(243y^5)$

$= 1024x^5 + 3840x^4y + 5760x^3y^2 + 4320x^2y^3 + 1620xy^4 + 243y^5$

13. Use the binomial theorem to expand.

c) $(0.3 + 0.7)^6$

Example:
$(a + b)^3 = {}_3C_0a^3 + {}_3C_1a^2b + {}_3C_2ab^2 + {}_3C_3b^3$
$= a^3 + 3a^2b + 3ab^2 + b^3$

c) $= {}_6C_0(0.3)^6(0.7)^0 + {}_6C_1(0.3)^5(0.7)^1 + {}_6C_2(0.3)^4(0.7)^2 + {}_6C_3(0.3)^3(0.7)^3 + {}_6C_4(0.3)^2(0.7)^4 + {}_6C_5(0.3)^1(0.7)^5 + {}_6C_6(0.3)^0(0.7)^6$

$= 1(0.000729) + 6(0.001701) + 15(0.003969) + 20(0.009261) + 15(0.021609) + 6(0.050421) + 1(0.117649)$

$= 0.000729 + 0.010206 + 0.059535 + 0.18522 + 0.324135 + 0.302526 + 0.117649$

$= 1$

13. Use the binomial theorem to expand.

d) $\left(\frac{1}{4}+\frac{3}{4}\right)^5$

Example:

$$(a+b)^3 = {}_3C_0a^3 + {}_3C_1a^2b + {}_3C_2ab^2 + {}_3C_3b^3$$
$$= a^3 + 3a^2b + 3ab^2 + b^3$$

d) $= {}_5C_0(1/4)^5(3/4)^0 + {}_5C_1(1/4)^4(3/4)^1 + {}_5C_2(1/4)^3(3/4)^2 + {}_5C_3(1/4)^2(3/4)^3 + {}_5C_4(1/4)^1(3/4)^4 + {}_5C_5(1/4)^0(3/4)^5$

$= 1(1/1024) + 5(3/1024) + 10(9/1024) + 10(27/1024) + 5(81/1024) + 1(243/1024)$

$= 1/1024 + 15/1024 + 90/1024 + 270/1024 + 405/1024 + 243/1024$

$= 1$