Unit 7 Review

Sequences, Series and Financial Applications

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Topics:

- Arithmetic Sequences
- Geometric Sequences
- Recursive Sequences
- Arithmetic and Geometric Series
- Pascal's Triangle and the Binomial Theorem
- Simple and Compound Interest
- Future Value and Present Value Annuities

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Solutions

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1. Determine S_{21} for the series $2.8 + 3.2 + 3.6 + 4.0 + \ldots$

a) 142.8 c) 10.8
b) 104 d) 142.4

$d = 0.4$, $a = 2.8$ $\Rightarrow$ arithmetic

$$S_n = \frac{n}{2}(2a + d(n-1))$$

$$S_{21} = \frac{21}{2}(2(2.8) + 0.4(21-1))$$

$$= 10.5(5.6 + 8$$

$$= 10.5(13.6)$$

$$= 142.8$$

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2. Identify the sequence that is not geometric.
   a) 4, 16, 64, 256, …
   b) 30, 6, 1.2, 0.24, …
   c) 2, 6, 7, 21, 22, …
   d) 5, 5, 5, 5, …

a) $r = \frac{16}{4} = \frac{64}{16} = \frac{256}{64} = 4$

b) $r = \frac{6}{30} = \frac{1.2}{6} = \frac{0.24}{1.2} = 0.2$

c) Multiply by 3, add 1, multiply by 3, add 1

d) $r = \frac{5}{5} = \frac{5}{5} = \frac{5}{5} = 1$

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3. Consider the sequence $1, -\frac{2}{3}, \frac{4}{9}, -\frac{8}{27}, \ldots$. Determine t_8.
   a) $\frac{128}{2187}$
   b) $\frac{64}{79}$
   c) $-\frac{128}{2187}$
   d) $-\frac{64}{79}$

$\frac{1}{1}, -\frac{2}{3}, \frac{4}{9}, -\frac{8}{27} \Rightarrow \frac{(-2)^{n-1}}{3^{n-1}}$

$\Rightarrow t_8 = \frac{(-2)^{8-1}}{3^{8-1}}$

$= \frac{(-2)^7}{3^7} = \frac{-128}{2187}$

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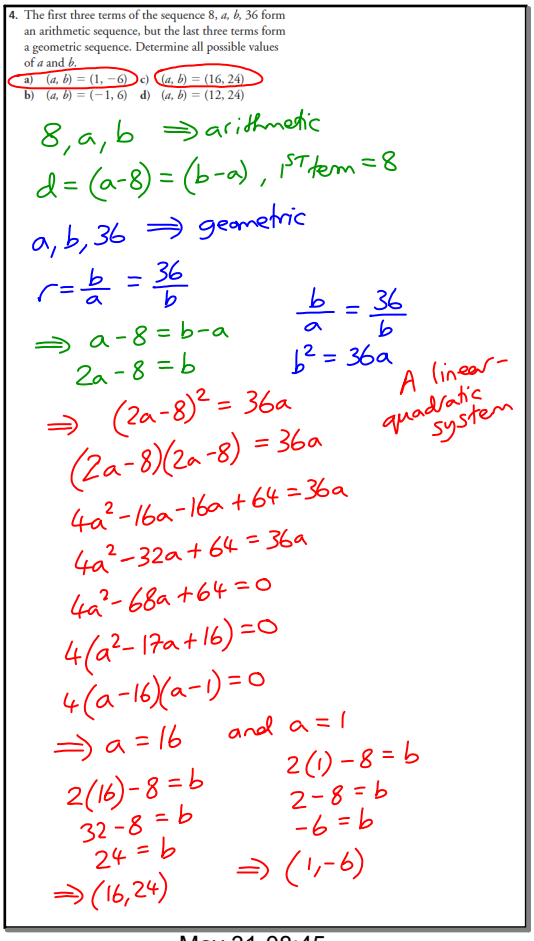

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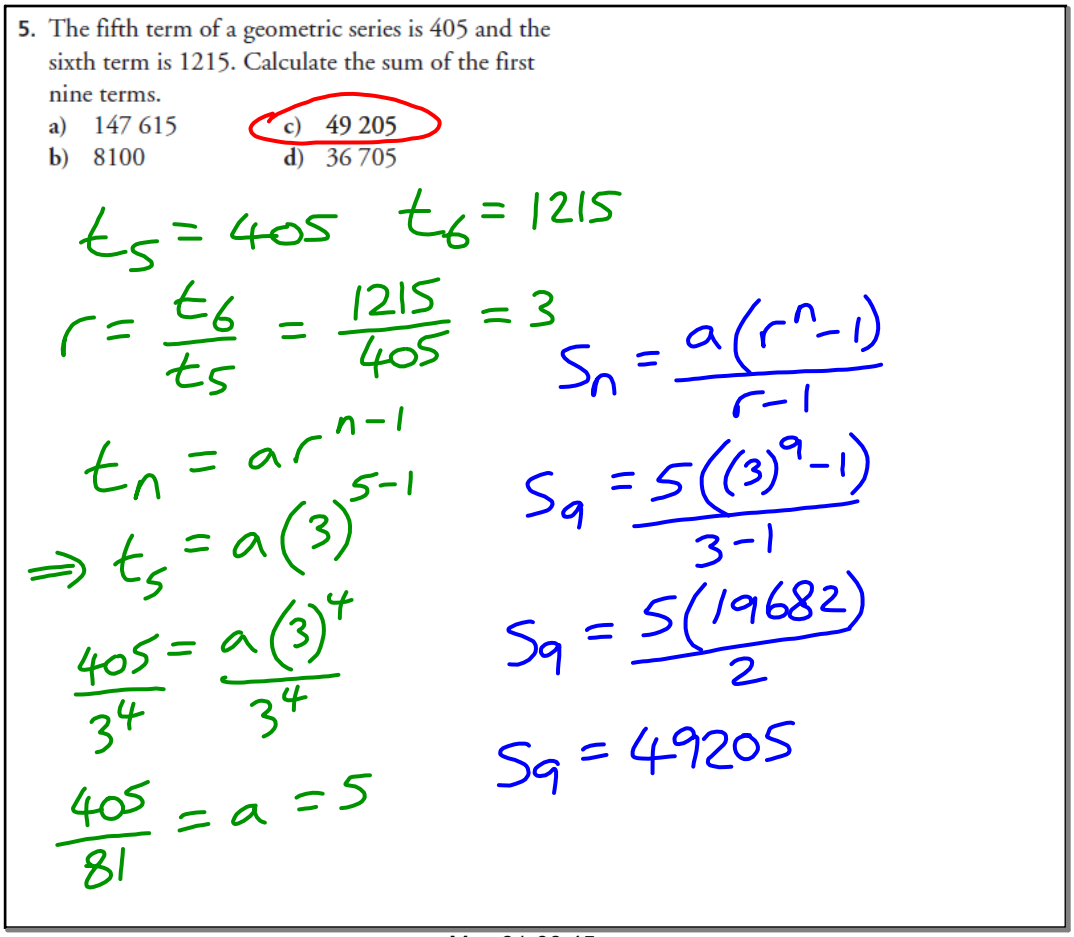

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6. Determine the first six terms of the sequence defined by $t_1 = -5$ and $t_n = -3t_{n-1} + 8$.

a) $-5, 23, -61, 191, -565, 1703$
b) $-5, 26, -70, 202, -598, 1786$
c) $-5, 23, 61, 191, 565, 1703$
d) $-5, 9, -51, 129, -411, 1209$

$t_1 = -5$

$t_2 = -3(-5) + 8$
$= 15 + 8$
$= 23$

$t_3 = -3(23) + 8$
$= -69 + 8$
$= -61$

$t_4 = -3(-61) + 8$
$= 183 + 8$
$= 191$

$t_5 = -3(191) + 8$
$= -573 + 8$
$= -565$

$t_6 = -3(-565) + 8$
$= 1695 + 8$
$= 1703$

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7. Choose the correct simplified expansion for the binomial $(x - 3)^5$.

a) $x^5 - 15x^4 + 90x^3 - 270x^2 + 405x - 243$
b) $x^4 - 15x^3 + 90x^2 - 270x + 405$
c) $x^6 - 15x^5 + 90x^4 - 270x^3 + 405x^2 - 243x$
d) $x^5 - 15x^4 + 90x^3 - 270x^2 + 405x$

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1
1 5 10 10 5 1

$= 1(x)^5(-3)^0 + 5(x)^4(-3)^1 + 10(x)^3(-3)^2$
$+ 10(x)^2(-3)^3 + 5(x)^1(-3)^4 + 1(x)^0(-3)^5$

$= x^5 - 15x^4 + 90x^3 - 270x^2 + 405x - 243$

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8. After 15 days, 90% of a radioactive material has decayed. What is the half-life of the material?

a) 1.45 days
b) 4.52 days
c) 7.5 days
d) 11.45 days

$A = A_0\left(\frac{1}{2}\right)^{t/h}$

$A = 0.1$
$A_0 = 1$
$t = 15$
$h = ?$

$0.1 = 1\left(\frac{1}{2}\right)^{15/h}$

$0.1 = \left(\frac{1}{2}\right)^{n}$ where $n = \frac{15}{h}$

Using guess and check....

$n = 3.32$

$\Rightarrow 3.32 = \frac{15}{h}$

$h = \frac{15}{3.32} = 4.52$ days

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9. Determine the annual interest rate, compounded annually, that would result in an investment doubling in seven years.

a) 10.4%
b) 14%
c) 7%
d) 11.45%

$A = P(1+i)^n$

$P = 1$
$A = 2$
$i = ?$
$n = 7$

$2 = 1(1+i)^7$

$2 = x^7$ where $x = 1+i$

$\sqrt[7]{2} = x$

$x = 1.104$

$\Rightarrow 1+i = 1.104$

$i = 0.104$

% rate $= 100i = 10.4\%$

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10. How long will it take for $5000 invested at 6%/a compounded monthly to grow to $6546.42?

a) 4.5 years
b) 3 years
c) 40 months
d) 48 months

$A = P(1+i)^n$

$P = 5000$

$A = 6546.42$

$i = \frac{0.06}{12}$

$n = ?$

$\frac{6546.42}{5000} = \frac{5000(1+\frac{0.06}{12})^n}{5000}$

$1.309284 = (1.005)^n$

Using guess and check

$n = 54$ months

$\Rightarrow \frac{54}{12} = 4.5$ years

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11. Marisa has just won a contest. She must decide between two prize options.

- Collect a lump-sum payment of $50 000.
- Receive $800 at the end of every quarter for 10 years from an investment.

The investment earns 8%/a, compounded quarterly. How much more money would she have if she chooses the lump sum?

a) $3009.89
b) $1678.41
c) $348.92
d) $30.99

$PV = \frac{R[1-(1+i)^{-n}]}{i}$

$R = 800$

$i = \frac{0.08}{4} = 0.02$

$n = 10(4) = 40$

$PV = ?$

$= \frac{800[1-(1+0.02)^{-40}]}{0.02}$

$= 21884.38$

Not a good question!

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12. An annuity written as a geometric series has r equal to 1.005. Determine the annual interest rate for the annuity if the interest is compounded monthly.

a) 12%
b) 0.5%
c) 5%
d) 6%

$1.005 = 1 + i$

$0.005 = i$

rate = i x compounding #

$= 0.005(12)$

$= 0.06$

$\Rightarrow$ % rate $= 0.06(100)$

$= 6\%$

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13. Lee wants to buy a plasma television. The selling price is \$1894. The finance plan includes \$150 down with payments of \$113 at the end of each month for $1\frac{1}{2}$ years. Determine the annual interest rate being charged, if the interest is compounded monthly.

a) 3.25%
b) 1%
c) 20.06%
d) 24%

$$PV = \frac{R\left[1-(1+i)^{-n}\right]}{i}$$

Need to use the finance app!

$PV = 1894 - 150$

$= 1744$

$R = 113$

$i = ?$

$n = 18$

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14. Mr. Los is planning to buy a sailboat. He decides to deposit $300 at the end of each month into an account that earns 6%/a interest, compounded monthly. At the end of four years, he uses the balance in the account as a down payment on a $56 000 sailboat. He gets financing for the balance at a rate of 8%/a, compounded monthly. He can afford payments of $525 per month. If interest rates remain constant, how long will it take him to repay the loan?

a) 10 years and 6 months
b) 12 years and 9 months
c) 9 years
d) 8 years and 10 months

$FV = R\left[\frac{(1+i)^n - 1}{i}\right]$

$FV = 300\left[\frac{\left(1+\frac{0.06}{12}\right)^{4(12)} - 1}{\frac{0.06}{12}}\right]$

$FV = 300\left[\frac{(1.005)^{48} - 1}{0.005}\right]$

$FV = \$16229.35$

Needs to borrow
$56000 - 16229.35$
$= \$39770.65$

$PV = 525\left[\frac{1-\left(1+\frac{0.08}{12}\right)^{-n}}{\frac{0.08}{12}}\right]$

$\frac{39770.65\left(\frac{0.08}{12}\right)}{525} = 1 - (1.006\bar{6})^{-n}$

$0.505024\ldots = 1 - x$
where $x = (1.006\bar{6})^{-n}$

$\Rightarrow x = 0.494975873$

$(1.006\bar{6})^{-n} = 0.49497\ldots$

Using guess and check
$n = 105.8$ months

$\Rightarrow \frac{106}{12} = 8.82$ years
$= 8$ years 10 months

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