Unit 4 Review

Trigonometric Ratios

Topics:

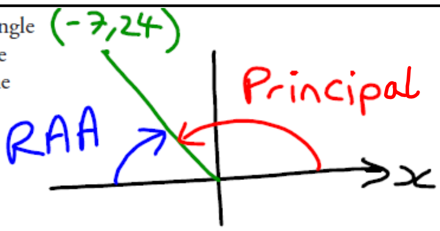
- Acute Triangles
- Special Angles
- Angle Terminology
- Extending Trig beyond 90 (Unit Circle)
- Trigonometric Identities
- Sine and Cosine Laws
- The Ambiguous Case
- 3D Trig Problems

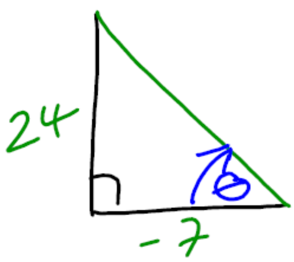
Nelson Page 408 #s 5 - 14 & 21 - 25

Solutions

5. Point $P(-7, 24)$ is on the terminal arm of an angle in standard position. What is the measure of the related acute angle and the principal angle to the nearest degree?

a) 74° and 106° c) 16° and 164°
b) 16° and 344° d) 74° and 286°





SOHCAHTOA

$\tan\theta = \frac{opp}{adj}$

$\tan\theta = \frac{24}{-7}$

$\theta = \tan^{-1}\left(\frac{24}{-7}\right) = -74°$

direction of turn

$\Rightarrow$ RAA $= 74°$

Principal $= 180 - 74 = 106°$

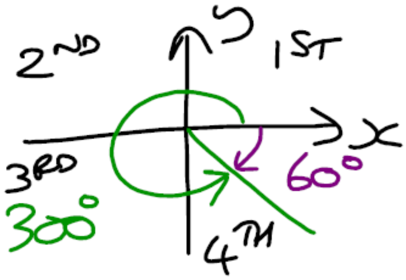
6. What is the exact value of csc 300°?

a) $\frac{\sqrt{3}}{2}$ b) $\frac{2}{\sqrt{3}}$ c) $-\frac{2\sqrt{3}}{3}$ d) $\frac{1}{2}$

COSECANT $= \frac{1}{\text{SIN}}$

SECANT $= \frac{1}{\text{COS}}$

COTANGENT $= \frac{1}{\text{TAN}}$



$$\csc 300 = \frac{1}{\sin 300} = -\frac{1}{\sin 60}$$

$$= -\frac{1}{\frac{\sqrt{3}}{2}}$$

$$= -\frac{2}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$$

$$= \frac{-2\sqrt{3}}{3}$$

7. Which equation is not an identity?

a) $(1 - \tan^2\theta)(1 - \cos^2\theta) = \frac{\sin^2\theta - 2\sin^4\theta}{1 - \sin^2\theta}$

b) $\frac{\tan x \sin x}{\tan x + \sin x} = \frac{\tan x \sin x}{\tan x \sin x}$

c) $\frac{\cos^2\theta - \sin^2\theta}{\cos^2\theta + \sin\theta\cos\theta} = 1 - \tan\theta$

d) $\frac{\frac{1 - \cos x}{\cos x}}{\tan x} = \frac{1 - \cos x}{\sin x}$

a) $(1 - \tan^2\theta)(1 - \cos^2\theta)$

$= 1 - \cos^2\theta - \tan^2\theta + \cos^2\theta\tan^2\theta$

$= 1 - \cos^2\theta - \frac{\sin^2\theta}{\cos^2\theta} + \cancel{\cos^2\theta}\left(\frac{\sin^2\theta}{\cancel{\cos^2\theta}}\right)$

$= \sin^2\theta + \sin^2\theta - \frac{\sin^2\theta}{\cos^2\theta}$

$= 2\sin^2\theta - \frac{\sin^2\theta}{\cos^2\theta}$

$= \frac{2\sin^2\theta\cos^2\theta - \sin^2\theta}{\cos^2\theta}$

$= \frac{2\sin^2\theta(1 - \sin^2\theta) - \sin^2\theta}{1 - \sin^2\theta}$

$= \frac{2\sin^2\theta - 2\sin^4\theta - \sin^2\theta}{1 - \sin^2\theta}$

$= \frac{\sin^2\theta - 2\sin^4\theta}{1 - \sin^2\theta} = RS$

7. Which equation is not an identity?

a) $(1 - \tan^2\theta)(1 - \cos^2\theta) = \dfrac{\sin^2\theta - 4\sin^4\theta}{1 - \sin^2\theta}$

b) $\dfrac{\tan x \sin x}{\tan x + \sin x} = \dfrac{\tan x \sin x}{\tan x \sin x}$

c) $\dfrac{\cos^2\theta - \sin^2\theta}{\cos^2\theta + \sin\theta\cos\theta} = 1 - \tan\theta$

d) $\dfrac{\frac{1 - \cos x}{\cos x}}{\tan x} = \dfrac{1 - \cos x}{\sin x}$

$$\frac{\cos^2\theta - \sin^2\theta}{\cos^2\theta + \sin\theta\cos\theta}$$

$$\frac{\cancel{(\cos\theta + \sin\theta)}(\cos\theta - \sin\theta)}{\cos\theta\,\cancel{(\cos\theta + \sin\theta)}}$$

$$\frac{\cos\theta - \sin\theta}{\cos\theta}$$

$$= \frac{\cos\theta}{\cos\theta} - \frac{\sin\theta}{\cos\theta} = 1 - \tan\theta = RS$$

7. Which equation is not an identity?

a) $(1 - \tan^2\theta)(1 - \cos^2\theta) = \dfrac{\sin^2\theta - 4\sin^4\theta}{1 - \sin^2\theta}$

b) $\dfrac{\tan x \sin x}{\tan x + \sin x} = \dfrac{\tan x \sin x}{\tan x \sin x}$

c) $\dfrac{\cos^2\theta - \sin^2\theta}{\cos^2\theta + \sin\theta\cos\theta} = 1 - \tan\theta$

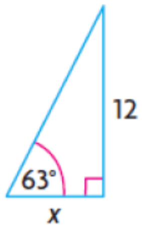
d) $\dfrac{\frac{1 - \cos x}{\cos x}}{\tan x} = \dfrac{1 - \cos x}{\sin x}$

$$\frac{1 - \cos x}{\cos x} \div \tan x$$

$$\frac{1 - \cos x}{\cos x} \div \frac{\sin x}{\cos x}$$

$$\frac{1 - \cos x}{\cancel{\cos x}} \times \frac{\cancel{\cos x}}{\sin x} = \frac{1 - \cos x}{\sin x} = RS$$

8. What is the measure of x to the nearest unit?



SOHCAHTOA

a) 4 b) 5 c) 6 d) 7

$$\tan\theta = \frac{opp}{adj}$$

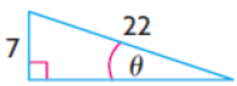
$$\tan 63 = \frac{12}{x}$$

$$\frac{x\tan 63}{\tan 63} = \frac{12}{\tan 63}$$

$$x = 6.1$$

9. What is the measure of θ to the nearest degree?

SOHCAHTOA



a) 19° b) 22° c) 15° d) 27°

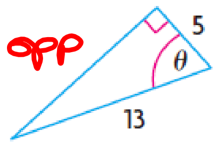
$$\sin\theta = \frac{opp}{hyp}$$

$$\sin\theta = \frac{7}{22}$$

$$\theta = \sin^{-1}\left(\frac{7}{22}\right)$$

$$\theta = 18.55°$$

10. Which is the correct ratio for csc θ?



a) $\frac{5}{13}$ b) $\frac{13}{5}$ c) $\frac{13}{12}$ d) $\frac{12}{5}$

$$\csc\theta = \frac{1}{\sin\theta} = \frac{1}{\left(\frac{opp}{hyp}\right)} = \frac{hyp}{opp}$$

$$\Rightarrow \csc\theta = \frac{13}{12}$$

Pythagorean theorem

$opp^2 + 5^2 = 13^2$

$opp^2 + 25 = 169$

$opp^2 = 169 - 25$

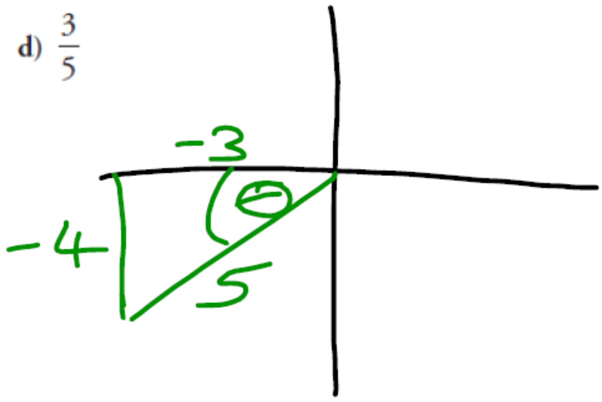
$opp^2 = 144$

$opp = \sqrt{144}$

$opp = 12$ cm

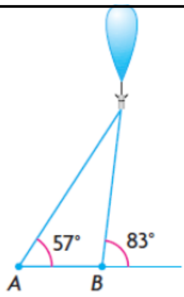
11. If $\tan\theta = \frac{4}{3}$ and θ lies in the third quadrant, which is the correct ratio for cos θ?

a) $\frac{4}{5}$ b) $-\frac{3}{5}$ c) $-\frac{4}{5}$ d) $\frac{3}{5}$

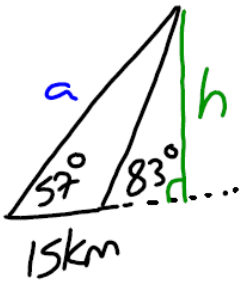


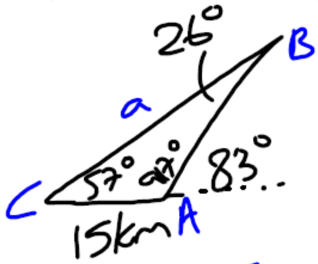
$$\cos\theta = \frac{adj}{hyp} = \frac{-3}{5}$$

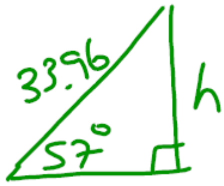
12. A weather balloon is spotted from two angles of elevation, 57° and 83°, from two different tracking stations. The tracking stations are 15 km apart. Determine the altitude of the balloon if the tracking stations and the point directly below the ballon lie along the same straight line.



a) 28.5 km
b) 32 km
c) 984 km
d) 23.7 km







$$\frac{a}{\sin 97} = \frac{15}{\sin 26}$$

$$a = \frac{15 \sin 97}{\sin 26}$$

$$a = 33.96\text{ km}$$

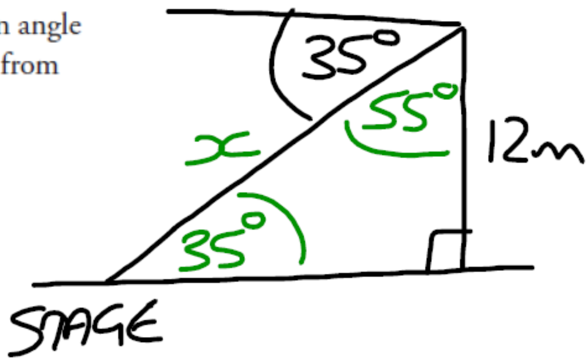
$$\sin 57 = \frac{h}{33.96}$$

$$33.96 \sin 57 = h$$

$$h = 28.48\text{ km}$$

13. At a concert, a spotlight is placed at a height of 12.0 m. The spotlight beam shines down at an angle of depression of 35°. How far is the spotlight from the stage?

a) 20.9 m
b) 12.1 m
c) 25 m
d) 9.6 m



SOHCAHTOA

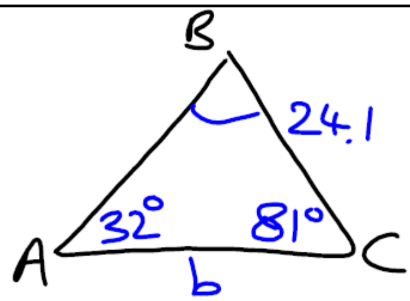
$$\sin 35 = \frac{12}{x}$$

$$\frac{x \sin 35}{\sin 35} = \frac{12}{\sin 35}$$

$$x = 20.9\text{ m}$$

14. In $\triangle ABC$, $\angle A = 32°$, $\angle C = 81°$, and $a = 24.1$. Solve the triangle, and identify the correct solution.

a) $\angle B = 125°$, $AC = 14.2$, $AB = 44.9$
b) $\angle B = 52°$, $AC = 41.9$, $AB = 44.9$
c) $\angle B = 107°$, $AC = 29.4$, $AB = 44.9$
d) $\angle B = 67°$, $AC = 41.9$, $AB = 44.9$



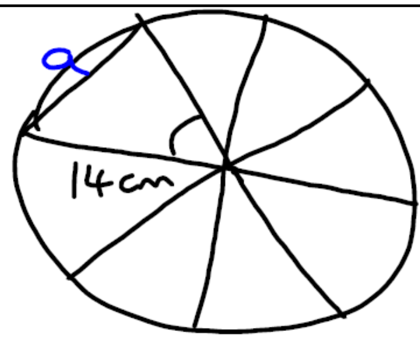
$$\angle B = 180 - 32 - 81$$
$$= 67°$$

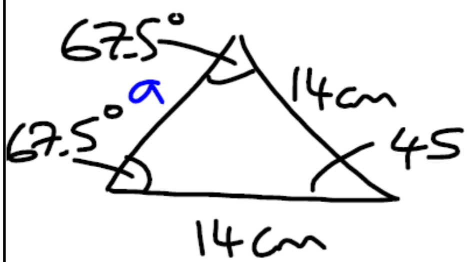
$$\frac{b}{\sin 67} = \frac{24.1}{\sin 32}$$
$$b = \frac{24.1 \sin 67}{\sin 32}$$
$$b = 41.86$$

$$\frac{c}{\sin 81} = \frac{24.1}{\sin 32}$$
$$c = \frac{24.1 \sin 81}{\sin 32}$$
$$c = 44.92$$

21. A regular octagon is inscribed inside a circle with a radius of 14 cm. The perimeter is

a) 32.9 cm
b) 56.0 cm
c) 85.7 cm
d) 42.9 cm



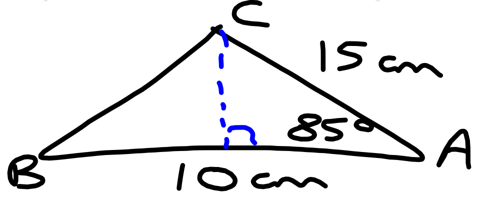


$$a^2 = b^2 + c^2 - 2bc\cos A$$
$$a^2 = 14^2 + 14^2 - 2(14)(14)\cos 45$$
$$a^2 = 114.814\ldots$$
$$a = \sqrt{ANS}$$
$$a = 10.715\ldots$$

$$\Rightarrow \text{Perimeter} = 8a$$
$$= 85.7\text{cm}$$

22. In $\triangle ABC$, $\angle A = 85°$, $c = 10$ cm, and $b = 15$ cm.
A possible height of $\triangle ABC$ is

a) 10.0 cm c) 13.8 cm
b) 8.6 cm d) 12.5 cm



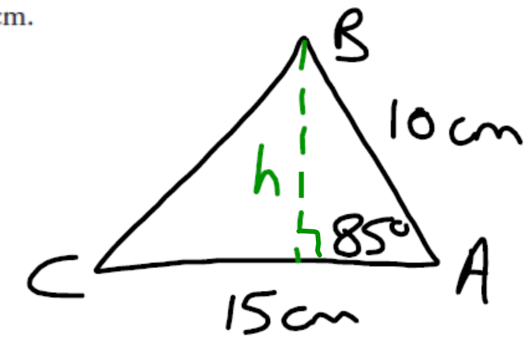
$\sin A = \frac{opp}{hyp}$

$\sin 85 = \frac{h}{15}$

$15 \sin 85 = h$

$14.9 = h$

$\Rightarrow$ Not one of the options



$\sin A = \frac{opp}{hyp}$

$\sin 85 = \frac{h}{10}$

$10 \sin 85 = h$

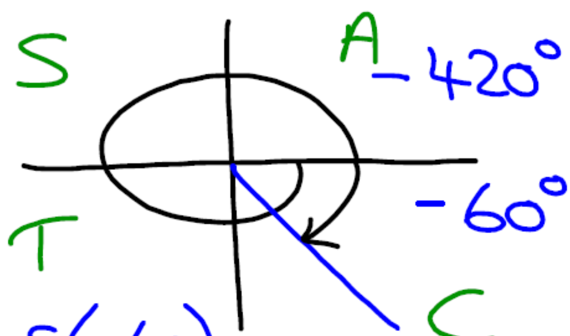
$9.96 = h$

$\Rightarrow$ 10.0 cm

23. The exact value of $\cos(-420°)$ is

a) $\frac{1}{2}$ c) $\frac{\sqrt{3}}{2}$

b) $-\frac{\sqrt{3}}{2}$ d) 1



$\cos(-420) = \cos(-60)$

$= \frac{1}{2}$

24. Using the definitions $\sin\theta = \frac{y}{r}$, $\cos\theta = \frac{x}{r}$, and $\tan\theta = \frac{y}{x}$, the simplified form of the expression $\dfrac{\sin^2\theta + \cos^2\theta}{\dfrac{\cos\theta}{\sin\theta}}$ is

a) $\frac{x}{y}$ b) $\frac{y}{x}$ c) $\frac{x}{r}$ d) $\frac{y}{r}$

$$\sin^2\theta + \cos^2\theta = 1$$

$$\frac{\sin\theta}{\cos\theta} = \tan\theta = \frac{y}{x}$$

$$\Rightarrow \frac{\cos\theta}{\sin\theta} = \frac{1}{(y/x)} = \frac{x}{y}$$

$$\frac{\sin^2\theta + \cos^2\theta}{\left(\frac{\cos\theta}{\sin\theta}\right)} = \frac{1}{\left(\frac{x}{y}\right)} = \frac{y}{x}$$

25. The simplified form of the expression $\dfrac{\sin x \sin x}{(1 - \sin x)(1 + \sin x)}$ is

a) $\dfrac{\sin^2 x}{\cos x}$ b) $\dfrac{\sin^2 x}{\sin x}$ c) $\tan^2 x$ d) $\dfrac{\sin^2 x}{1 + \sin^2 x}$

$$= \frac{\sin^2 x}{(1 + \cancel{\sin x} - \cancel{\sin x} - \sin^2 x)}$$

$$= \frac{\sin^2 x}{1 - \sin^2 x}$$

$$= \frac{\sin^2 x}{\cos^2 x} = \tan^2 x$$