Unit 1 Review

Quadratic Functions

Topics:

- Properties of Quadratic Functions
- Forms of a Quadratic Equation
- Review of Factoring
- Zeros of a Quadratic
- Determining Max and Min Values
- Problem Solving with Quadratics
- Writing Quadratic Equations
- Linear-Quadratic Systems
- Transformations

Nelson Page 206 #s 9 - 14 & 26 - 30

Solutions

9. Given the quadratic function $f(x) = 3x^2 - 6x + 15$, identify the coordinates of the vertex.

a) (1, 12) c) (12, 1)
b) (−1, −12) d) (12, −1)

Either complete the square

$3(x^2 - 2x) + 15$

$3(x^2 - 2x + (-1)^2 - (-1)^2) + 15$

$3(x^2 - 2x + (-1)^2) - (3)(-1)^2 + 15$

$3(x-1)^2 - 3 + 15$

$3(x-1)^2 + 12$

$\Rightarrow$ Vertex $= (1, 12)$

OR

Use $\left(\frac{-b}{2a}, f\left(\frac{-b}{2a}\right)\right)$

$\Rightarrow -\frac{b}{2a} = \frac{6}{2(3)}$

$= \frac{6}{6} = 1$

$f(1) = 3(1)^2 - 6(1) + 15$

$= 3 - 6 + 15$

$= 12$

$\Rightarrow$ Vertex $= (1, 12)$

10. When the equation of a quadratic function is in factored form, which feature is most easily determined?

a) *y*-intercepts
b) *x*-intercepts
c) vertex
d) maximum value

$f(x) = a(x-r)(x-s)$

x-intercepts are r and s

11. The height, *h*, in metres, of a baseball after Bill hits it with a bat is described by the function $h(t) = 0.8 + 29.4t - 4.9t^2$, where *t* is the time in seconds after the ball is struck. What is the maximum height of the ball?

a) 4.9 m b) 29.4 m c) 44.9 m d) 25 m

Using $\frac{-b}{2a} = \frac{-29.4}{2(-4.9)}$

$= \frac{-29.4}{-9.8}$

$= 3$

$\Rightarrow h(3) = 0.8 + 29.4(3) - 4.9(3)^2$

$= 44.9\text{m}$

12. It costs a bus company $225 to run a minibus on a ski trip, plus $30 per passenger. The bus has seating for 22 passengers, and the company charges $60 per fare if the bus is full. For each empty seat, the company has to increase the ticket price by $5. How many empty seats should the bus run with to maximize profit from this trip?
a) 8 b) 6 c) 10 d) 2

Cost = $225 + 30(22 - x)$

Revenue = $(60 + 5x)(22 - x)$

where x = # of empty seats

Profit = Revenue - Cost

$= (60 + 5x)(22 - x) - (885 - 30x)$

$= 1320 - 60x + 110x - 5x^2 - 885 + 30x$

$= -5x^2 + 80x + 435$

Max profit when $x = \frac{-b}{2a}$

$x = \frac{-80}{2(-5)}$

$x = \frac{-80}{-10} = 8$ empty seats

13. Without drawing the graph, identify the function that has two zeros.
a) $n(x) = -x^2 - 6x - 9$
b) $m(x) = 4(x + 1)^2 + 0.5$
c) $h(x) = -5(x + 1.3)^2$
d) $g(x) = -2(x + 3.6)^2 + 4.1$

From standard form find the discriminant

$b^2 - 4ac$

$\Rightarrow (-6)^2 - 4(-1)(-9)$

$= 36 - 36$

$= 0$

$\Rightarrow$ 1 zero

From vertex form look at the signs of a and k

Different $\Rightarrow$ 2 zeros

Same $\Rightarrow$ 0 zeros

$k = 0 \Rightarrow$ 1 zero

(d) a is negative
k is positive

14. The graph of function $f(x) = x^2 - kx + k + 8$ touches the x-axis at one point. What are the possible values of k?

a) $k = 1$ or $k = 8$
b) $k = -4$ or $k = 8$
c) $k = 0$ or $k = 1$
d) $k = -8$ or $k = 4$

Think of it as $f(x) = x^2 - kx + (k+8)$

where $a=1, b=-k, c=k+8$

For one zero $\Rightarrow b^2 - 4ac = 0$

$(-k)^2 - 4(1)(k+8) = 0$

$k^2 - 4k - 32 = 0$

Find values of k that make brackets equal zero $\rightarrow (k-8)(k+4) = 0$

$\Rightarrow k = 8, k = -4$

26. The vertex form of the equation $y = -2x^2 - 12x - 19$ is

a) $y = -2x(x + 6) - 19$
b) $y = -2(x - 3)(x + 6)$
c) $y = -2(x + 3)^2 - 1$
d) $y = -2(x - 3)^2 + 1$

$\frac{-b}{2a} = \frac{12}{2(-2)} = \frac{12}{-4} = -3$

$f(-3) = -2(-3)^2 - 12(-3) - 19$

$= -18 - (-36) - 19$

$= -1$

$y = -2(x^2 + 6x) - 19$

$y = -2(x^2 + 6x + (3)^2 - (3)^2) - 19$

$y = -2(x^2 + 6x + (3)^2) - (-2)(3)^2 - 19$

$y = -2(x+3)^2 - (-18) - 19$

$y = -2(x+3)^2 - 1$

$\Rightarrow y = -2(x+3)^2 - 1$

27. The coordinates of the vertex for the graph of $y = (x + 2)(x - 3)$ are

a) $(-2, 3)$ c) $(2, 3)$

b) $\left(-\frac{1}{2}, -\frac{21}{4}\right)$ d) $\left(\frac{1}{2}, -\frac{25}{4}\right)$

$y = (x+2)(x-3)$

$\Rightarrow r = -2, s = 3$

$\text{Vertex} = \left(\frac{r+s}{2}, f\left(\frac{r+s}{2}\right)\right)$

$= \left(\frac{1}{2}, -6\frac{1}{4}\right)$

$= \left(\frac{1}{2}, -\frac{25}{4}\right)$

$\frac{r+s}{2} = \frac{-2+3}{2}$

$= \frac{1}{2}$

$f\left(\frac{1}{2}\right) = \left(\frac{1}{2}+2\right)\left(\frac{1}{2}-3\right)$

$= (2.5)(-2.5)$

$= -6.25$

28. The profit function for a new product is given by $P(x) = -4x^2 + 28x - 40$, where x is the number sold in thousands. How many items must be sold for the company to break even?

a) 2000 or 5000 c) 5000 or 7000

b) 2000 or 3500 d) 3500 or 7000

Break even when profit = 0

$\Rightarrow 0 = -4x^2 + 28x - 40$

$0 = -4(x^2 - 7x + 10)$

$0 = -4(x-5)(x-2)$

$\Rightarrow x = 5, x = 2$

$\Rightarrow$ # of items = 2000 or 5000

29. Which of the following statements is not true for the equation of a quadratic function?

a) In standard form, the y-intercept is clearly visible.

b) In vertex form, the break-even points are clearly visible.

c) In factored form, the x-intercepts are clearly visible.

d) In vertex form, the coordinates of the vertex are clearly visible.

$y = ax^2 + bx + c$

$y = a(x-h)^2 + k$

$y = a(x-r)(x-s)$

$y = a(x-h)^2 + k$

30. State the value of the discriminant, D, and the number of roots for $7x^2 + 12x + 6 = 0$.

a) $D = 312, n = 2$

b) $D = 24, n = 2$

c) $D = 312, n = 1$

d) $D = -24, n = 0$

$b^2 - 4ac$

$\Rightarrow (12)^2 - 4(7)(6)$

$= 144 - 168$

$= -24$ ← negative $\Rightarrow$ 0 zeros

0 $\Rightarrow$ 1 zero

positive $\Rightarrow$ 2 zeros