Review

Probability Distributions for Discrete Variables

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- A probability distribution shows the probabilities of all possible outcomes in an experiment.
- The sum of all probabilities in any distribution is 1.
- A probability histogram graphs the relative frequency of the random variable. The area of each bar represents the probability of the variable.
- Expectation, or expected value, is the weighted average value of the random variable.

$$E(X) = x_1 \cdot P(x_1) + x_2 \cdot P(x_2) + \cdots + x_n \cdot P(x_n)$$
$$= \sum_{i=1}^{n} x_i \cdot P(x_i)$$

The expectation can be a non-integer value.

- A uniform distribution occurs when, in a single trial, all outcomes are equally likely.
- For a uniform distribution, $P(x) = \frac{1}{n}$, where n is the number of possible outcomes in the experiment.
- When calculating expectation for a uniform distribution, you can factor $\frac{1}{n}$ to make the calculations easier: $E(X) = \frac{1}{n}\sum_{i=1}^{n} x_i$
- When calculating expectation, you can calculate the sum of the numbers from 1 to n using the expression $\frac{n(n+1)}{2}$.
- The expected outcome of a fair game is equal to 0.

- A binomial distribution has a specific number of identical independent trials in which the result is success or failure.
- You can represent a binomial distribution using a table, a histogram, and a formula.
- The probability of x successes in n independent trials is $P(x) = {}_nC_x p^x q^{n-x}$, where p is the probability of success in an individual trial, and $q = 1 - p$ is the probability of failure.
- The expectation for the binomial distribution is $E(X) = np$.
- A hypergeometric probability distribution occurs when there are two outcomes, success and failure, and all trials are dependent. The random variable is the number of successes in a given number of trials.
- You can represent a hypergeometric distribution using a table, a probability histogram, or a formula.
- The probability of x successes in r dependent trials is $P(x) = \frac{{}_aC_x \cdot {}_{n-a}C_{r-x}}{{}_nC_r}$, where a is the number of successful outcomes available in a population of size n.
- Expectation $E(X) = \frac{ra}{n}$.

- The chart summarizes the general conditions of the distributions.

	Uniform	Binomial	Hypergeometric
Parameters and What They Represent	n = number of items	n = number of trials p = probability of success on an individual trial q = probability of failure on an individual trial	n = size of the population r = number of trials a = number of successful items available
Definition of Random Variable, *x*	Value of the outcome	Number of successful outcomes	Number of successful outcomes
Range of Values for *x*	Depends on the situation	$x = 0, 1, 2, \ldots, n$	$x = 0, 1, 2, \ldots, r$
Probability Formula	$P(x) = \frac{1}{n}$	$P(x) = {}_nC_x p^x q^{n-x}$	$P(x) = \frac{{}_aC_x \cdot {}_{n-a}C_{r-x}}{{}_nC_r}$
Expectation Formula	$E(X) = \frac{1}{n}\sum_{i=1}^{n} x_i$	$E(X) = np$	$E(X) = \frac{ra}{n}$
Identifying Characteristics	All items are equally likely A single trial	Trials are independent Successes are counted	Trials are dependent Successes are counted

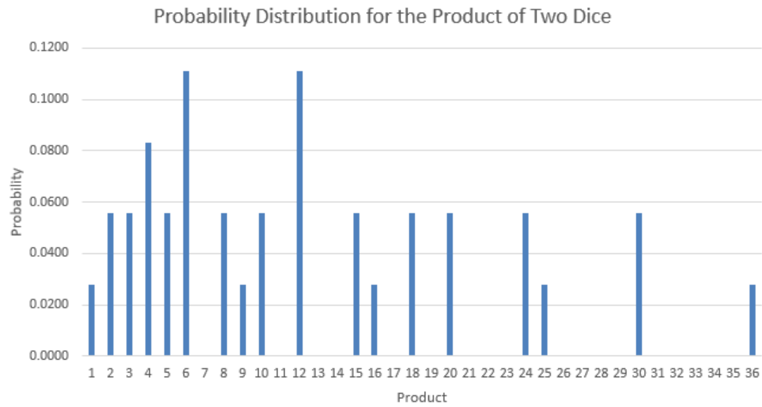
Solutions

1. Two dice are rolled and the product of the upper faces is recorded. Show the probability distribution in table form and graphically.

Product = Multiply

Product	Frequency	Probability
1	1	0.0278
2	2	0.0556
3	2	0.0556
4	3	0.0833
5	2	0.0556
6	4	0.1111
8	2	0.0556
9	1	0.0278
10	2	0.0556
12	4	0.1111
15	2	0.0556
16	1	0.0278
18	2	0.0556
20	2	0.0556
24	2	0.0556
25	1	0.0278
30	2	0.0556
36	1	0.0278

	1	2	3	4	5	6
1	1	2	3	4	5	6
2	2	4	6	8	10	12
3	3	6	9	12	15	18
4	4	8	12	16	20	24
5	5	10	15	20	25	30
6	6	12	18	24	30	36



2. A set of cards with the numbers 200 to 299 is used in a game. The cards are shuffled and the top card is turned up. Calculate the expectation and explain its meaning.

This is a uniform distribution because each card has an equal chance of being selected.

$E(x) = 0.01(200 + 201 + 202 + ... + 298 + 299)$

$= 0.01[50(499)]$

$= 249.5$

The expected value is 249.5

This means that the predicted value of the card that is selected is 249.5, and yes we know that this value can't actually turn up.

3. The serial numbers on $5 bills include three letters followed by seven digits. Assuming the digits are assigned at random, what is the probability that a serial number will contain

a) exactly two 5s?

b) at least four 5s?

c) all 5s?

$p = P(5) = 0.1$

$q = P(\text{Not } 5) = 0.9$

$n = 7$

a) P(Exactly two 5's) $= {}_7C_2(0.1)^2(0.9)^5$

$= 21(0.01)(0.59049)$

$= 0.2140...$

The probability of having exactly two 5's in the serial number is about 0.2140

b) P(At least four 5's) = P(4 5's) + P(5 5's) + P(6 5's) + P(7 5's)

$= {}_7C_4(0.1)^4(0.9)^3 + {}_7C_5(0.1)^5(0.9)^2 + {}_7C_6(0.1)^6(0.9)^1 + {}_7C_7(0.1)^7(0.9)^0$

$= 0.0025515 + 0.0001701 + 0.0000063 + 0.0000001$

$= 0.002728$

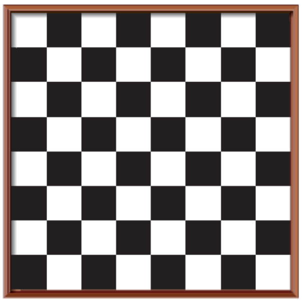
The probability of having at least four 5's in the serial number is about 0.0027

c) P(All 5's) $= {}_7C_7(0.1)^7(0.9)^0$

$= 0.0000001$

The probability of having all 5's in the serial number is 0.0000001 (1×10^{-7})

4. Five checkers are randomly placed on a checkerboard. What is the probability that three checkers are on squares of one colour and two checkers are on another colour?



64 squares on a board

32 are black and 32 are white

The 5 checkers can be placed in ${}_{64}C_5$ ways.

To have 3 on one colour is ${}_{32}C_3$ and 2 on the other colour is ${}_{32}C_2$

$$P(A) = \frac{{}_{32}C_3 \times {}_{32}C_2}{{}_{64}C_5}$$

$P(A) = 0.32266...$

The probability of 3 checkers being on one colour and 2 being on the other colour is about 0.3227

5. a) Show the probability distributions, in table form and graphically, for the following distributions:

i) Selecting a card four times, with replacement, from a standard deck, and recording the number of diamonds.

ii) Selecting four cards at the same time, from a standard deck, and recording the number of diamonds.

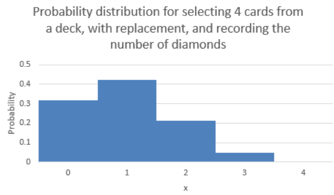
b) Compare the resulting graphs.

c) Compare the expectations and comment on your findings.

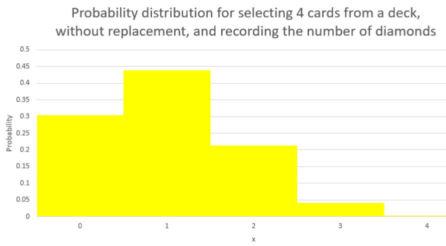
(i) Binomial distribution because the card is replaced

(ii) Hypergeometric distribution because the card is NOT replaced

x	p	q	nCx	P(x)
0	0.2500	0.7500	1	0.316406
1	0.2500	0.7500	4	0.421875
2	0.2500	0.7500	6	0.210938
3	0.2500	0.7500	4	0.046875
4	0.2500	0.7500	1	0.003906



x	n	r	a	n-a	r-x	aCx	n-aCr-x	nCr	P(x)
0	52	4	13	39	4	1	82251	270725	0.3038175
1	52	4	13	39	3	13	9139	270725	0.4388475
2	52	4	13	39	2	78	741	270725	0.2134934
3	52	4	13	39	1	286	39	270725	0.0412005
4	52	4	13	39	0	715	1	270725	0.0026411



b) Both graphs have a bell-like shape to them with the highest probability being one diamond. For 3 or 4 diamonds the hypergeometric distribution has a lower probability because there are fewer diamonds that can be chosen.

c) The expected value for the binomial distribution is np = 4(0.25) = 1

The expected value for the hypergeometric distribution is ra/n = 4(13)/52 = 1. So, on average we will expect to get one diamond from four selections, whether we replace the cards or not.