Review

Combinations

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- The number of permutations of n objects, when p of one type are identical, q of another type are identical, r of another type are identical, and so on, is $n(A) = \frac{n!}{p!\,q!\,r!\ldots}$.
- If a number of distinct objects need to remain in a specific order in a permutation, divide by the factorial of that number.
- A combination is a set of items taken from another set in which order does not matter. In a permutation, the order of the items matters.
- The number of combinations of r items taken from a set of n items is ${}_nC_r = \frac{n!}{(n-r)!\,r!}$.

- The total number of subsets of a set of n elements is ${}_nC_0 + {}_nC_1 + \ldots + {}_nC_n = 2^n$.
- In some cases the null set is not considered. In such cases, ${}_nC_1 + {}_nC_2 + \ldots + {}_nC_n = 2^n - 1$.
- Consider using the indirect method, especially if it involves fewer cases, such as when you need to choose at least one or two items.
- If the order is important, consider selecting the items first and then arranging them in order.

- The terms in row n of Pascal's triangle correspond to the combinations $t_{n,r} = {}_nC_r$.
- A given term in Pascal's triangle equals the sum of the two terms directly above it in the previous row. They can be generated using the relationship $t_{n,r} + t_{n,r+1} = t_{n+1,r+1}$. This is known as Pascal's method.
- Using combinations, ${}_nC_r + {}_nC_{r+1} = {}_{n+1}C_{r+1}$.
- The coefficients in the binomial expansion of $(x + y)^n$ are found in row n of Pascal's triangle.
- According to the binomial theorem, $(x + y)^n = {}_nC_0x^ny^0 + {}_nC_1x^{n-1}y^1 + {}_nC_2x^{n-2}y^2 + \ldots + {}_nC_{n-1}x^1y^{n-1} + {}_nC_nx^0y^n$.
- Pascal's method can be applied to counting paths in arrays.
- You can sometimes use combinations or Pascal's triangle to determine probabilities.
- The numerator, $n(A)$, represents the number of successful outcomes, usually involving restrictions.
- The denominator, $n(S)$, represents the total number of outcomes, with no restrictions.

Solutions

15. Before a school dance, students tweeted requests to the DJ for five hip hop, seven R&B, eight rock, and nine pop songs. The DJ will play three requested songs from each genre. How many different playlists could the DJ generate?

This like all of these types of questions will depend upon where order is important or not.

When choosing the three songs from each genre, the order of selection is not important, so we will use combinations.

Hip Hop = ${}_5C_3$

R&B = ${}_7C_3$

Rock = ${}_8C_3$

Pop = ${}_9C_3$

Number of playlists = ${}_5C_3 \times {}_7C_3 \times {}_8C_3 \times {}_9C_3$

= 1,646,400

16. To get to school, you travel six blocks west and four blocks south.

a) Use permutations, combinations, or Pascal's method to determine the number of routes you could take to school.

b) Relate your method to one of the other methods.

a) Need to travel a total of 10 blocks to school. 6 of these are west, so $_{10}C_6$ and the remaining 4 are south, so $_4C_4$. You have take both the west AND south parts of the journey so we are multiplying.

$_{10}C_6 \times {_4C_4} = 210 \times 1$

$= 210$ different routes to school

b) If using Pascal's method you need to add the number paths to the adjacent grid points to determine the number of paths to the given point.

17. Communication Copy the table and extend it by six rows.

a) Use a calculator or refer to Pascal's triangle to complete the chart up to $n = 9$.

n	nC2 ÷ nC1	Result
2	1 ÷ 2	0.5
3	3 ÷ 3	1
4	6 ÷ 4	1.5
5	10 ÷ 5	2
6	15 ÷ 6	2.5
7	21 ÷ 7	3
8	28 ÷ 8	3.5
9	36 ÷ 9	4

b) For which values of n is $_nC_2$ divisible by $_nC_1$?

c) Generalize your findings as they relate to combinations and Pascal's triangle.

d) Is $_{15}C_2$ divisible by $_{15}C_1$? How do you know, without actually calculating it?

b) $_nC_2$ is divisible by $_nC_1$ when n is odd.

c) When n is odd, $_nC_2$ is divisible by $_nC_1$. Rows of Pascal's triangle where n is odd have an even number of terms.

d) Yes it is. We know because n (15) is an odd number.

18. In how many ways can eight tickets be put into two envelopes if one envelope is to contain five tickets and the other envelope is to contain three tickets?

We are not concerned with which tickets go where, so order is not important.

First envelope has 5 tickets. They can be selected in ${}_8C_5$ ways.

Second envelope has 3 tickets. They can be selected in ${}_3C_3$ ways.

Total number of ways = ${}_8C_5 \times {}_3C_3$ = 56 ways.

19. How many different sums of money can be made from a \$5, a \$10, a \$20, a \$50, and a \$100 bill?

a) Use the direct method.

b) Use the indirect method.

a) Five different bills. We can choose any one, any two, any three, any four or all five bills to make a sum of money.

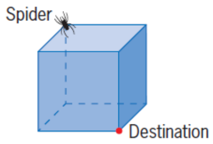
= 5C1 + 5C2 + 5C3 + 5C4 + 5C5

= 5 + 10 + 10 + 5 + 1

= 31 different sums

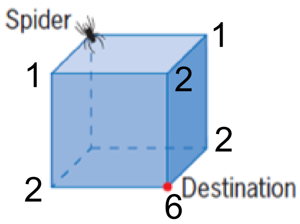
b) Five bills so we can choose or not choose each which is $2^5 = 32$ different sums. However, since we can not have zero we need to subtract the null set leaving 31 different sums.

20. A spider walks from one corner of a cube to the diagonally opposite corner.



a) If the spider walks along the edges only, and never backtracks, how many different paths can it take?

b) Can combinations be used to solve this problem? Why or why not?



a) The numbers represent the number of different paths to get to each corner. We can get the "6" by adding together the corners adjacent to the destination.

b) Combinations can be used. From the spider's perspective it has to travel along three edges that are either left, right, or down.

First choice is one from three edges = ${}_3C_1$

Second choice is one from two edges = ${}_2C_1$

Final choice is one from one edge = ${}_1C_1$

Total combinations = ${}_3C_1 \times {}_2C_1 \times {}_1C_1$

= 3 x 2 x 1 = 6 different paths.

21. A jury of 12 people is chosen from 20 men and 30 women. What is the probability that

a) there is an equal number of men and women on the jury?

b) there are at least two men on the jury?

a) There are ${}_{50}C_{12}$ possible jury combinations.

For an equal number of men we are selecting ${}_{20}C_6$ AND for an equal number of women we are selecting ${}_{30}C_6$.

P(6 Men and 6 Women) = $({}_{20}C_6 \times {}_{30}C_6) \div {}_{50}C_{12}$

= 0.189578131...

The probability of an equal number of men and women on the jury is about 0.1896.

b) Use the indirect method. Use all possible combinations and subtract arrangements for no men and arrangements for one man.

To have no men = ${}_{20}C_0 \times {}_{30}C_{12}$

To have one man = ${}_{20}C_1 \times {}_{30}C_{11}$

P(0 or 1 man) = $[({}_{20}C_0 \times {}_{30}C_{12}) + ({}_{20}C_1 \times {}_{30}C_{11})] \div {}_{50}C_{12}$

= 0.009712

P(At least 2 men) = 1 - 0.009712

= 0.990288

The probability of the jury having at least 2 men on it is 0.9903