Review

Permutations

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- You can illustrate a sequence of events using multiple methods, including a list, a chart, and a tree diagram.
- In a tree diagram, each stage in the event is illustrated with a new set of branches extending from the end of each branch in the previous stage.
- To identify a given outcome in a tree diagram, read across a distinct path.
- To determine the number of outcomes in a tree diagram, count the number of distinct end paths across the tree diagram.
- If one event can occur in m ways and a second event can occur in n ways, then together they can occur in $m \times n$ ways. This is called the fundamental counting principle.
- The fundamental counting principle can be used for multiple trials. For example, events can occur in $m \times n \times p$ … ways.

- The number of permutations of n items is n factorial,

  $$n! = n(n-1)(n-2)(n-3) \dots \times 3 \times 2 \times 1$$

- You can use factorials as a counting technique when repetition is not permitted.
- The number of r-permutations of n items can be calculated by

  $$_nP_r = n(n-1)(n-2) \cdots (n-r+1)$$
  $$= \frac{n!}{(n-r)!}$$

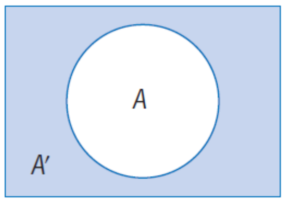
- The rule of sum states that if one mutually exclusive event can occur in m ways, and a second can occur in n ways, then one **or** the other can occur in $m + n$ ways.
- If two events are not mutually exclusive, the principle of inclusion and exclusion needs to be considered:

  $$n(A \text{ or } B) = n(A) + n(B) - n(A \text{ and } B).$$

- To reduce calculations, consider using the indirect method, which involves subtracting the unwanted event from the total number of outcomes in the sample space: $n(A) = n(S) - n(A')$.

- You can calculate the probability of an event using $P = \frac{n(A)}{n(S)}$, where $n(A)$ is the number of successful outcomes and $n(S)$ is the number of outcomes in the sample space.
- If the trials are dependent, you can use permutations in the calculations.
- To use the indirect method, subtract the probability of the complement from 1.

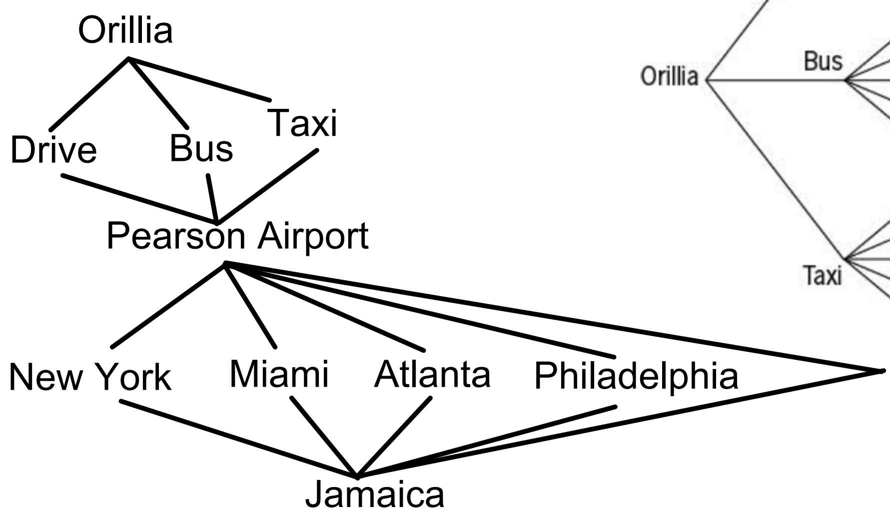
  $$P(A) = 1 - P(A')$$

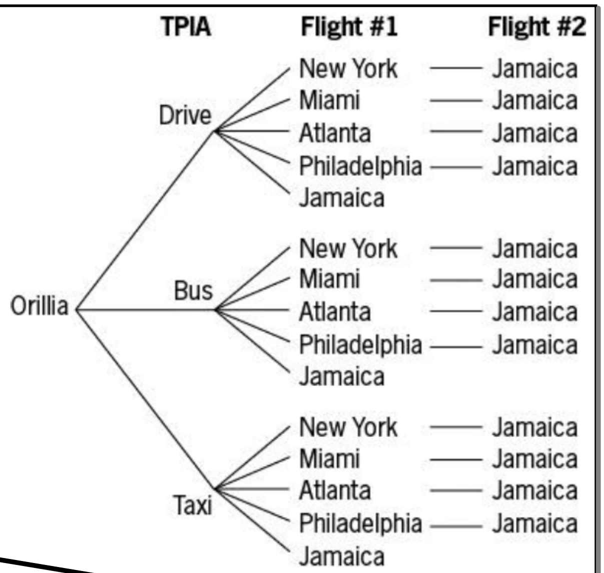


$P(A) = 1 - P(A')$

Solutions

7. Kaan lives in Orillia and will be flying to Jamaica on vacation. To get to Toronto Pearson International Airport, he can drive, take a bus, or take a taxi. He can fly non-stop to Jamaica or he can go via New York, Miami, Atlanta, or Philadelphia. Draw a map, a tree diagram, and make a list of all possible routes Kaan can take to Jamaica.





There are a total of 15 different routes to get from Orillia to Jamaica when travelling via Toronto Pearson International Airport.

8. How many different outcomes are there when rolling

a) three standard dice?

b) four standard dice?

c) two 8-sided dice?

d) three 12-sided dice?

6 outcomes on a standard die

a) 6 x 6 x 6 = 216 outcomes

b) 6 x 6 x 6 x 6 = 1296 outcomes

c) 8 x 8 = 64 outcomes

d) 12 x 12 x 12 = 1728 outcomes

9. a) In how many ways could four adjacent countries on a map be coloured if eight colours are available? Adjacent countries must be different colours.

b) Why is it important that the countries are adjacent?

a) We can use 8 colours for the first country, 7 colours for the second, 6 for the third and five for the fourth.

= 8(7)(6)(5) = 1680 ways

b) If the countries are adjacent it means we can not use a previously used colour. The reason for this is that these countries share a border, so to help to distinguish between them, different colours are used.

10. A total of 500 people enter a draw in which there is a first prize, a second prize, and a third prize. In how many ways could the prizes be awarded?

500 people could win the first prize, 499 the second and 498 the third.

Therefore there are 500(499)(498) = 124,251,000 ways

OR....

Use permutations where we want 3 "winners" from the 500 people. The order is important as the prizes are likely to be of different value.

${}_{500}P_3$ = 124,251,000 ways

11. In how many ways could a president, vice president, secretary, and treasurer be elected from a condominium board that has 8 members?

8 ways to select the president, 7 ways the vice-president, ...

= 8(7)(6)(5)

= 1680 ways

OR ...

Choosing 4 people from 8 where the order is important:

${}_8P_4$ = 1680 ways

12. In how many ways could 12 people be seated at a rectangular table if the two hosts must not sit together?

There are 12! ways to arrange everybody.

If the two hosts DO sit together there are 11! ways to seat everybody (treat the two hosts as one person).

Using the indirect method there are 12! - 11! ways to seat everybody if they are NOT sat together.

= 12! - 11!

= 479,001,600 - 39,916,800

= 439,084,800 ways

13. To win a school fundraising lottery, you need to correctly select five different digits in the correct order.

a) What is the probability of winning?

b) Would winning be more or less probable if the digits could be repeated? Why?

a) To select 5 digits in the correct order, there are 5! ways the digits could be ordered. There is only one correct way to select them.

P(5 digits in correct order) = 1 / 5!

= 1/120

b) If the digits could be repeated there would be 5 x 5 x 5 x 5 x 5 = 3125 ways they could be ordered. Again there is only one correct way to select them so.

P(5 digits in correct order) = 1/3125

Therefore winning would be LESS probable if the digits could be repeated.

14. Find the prime factors of 255 255. What is the total number of divisors of 255 255, excluding 1 and itself?

First find the prime factors of 255,255

$255{,}255 \div 3 = 85{,}085$

$85{,}085 \div 5 = 17{,}017$

$17{,}017 \div 7 = 2{,}431$

$2{,}431 \div 11 = 221$

$221 \div 13 = 17$

So 255,255 = 3 x 5 x 7 x 11 x 13 x 17 which is a total of 6 PRIME factors.

The total number of divisors are made up from all the possible numbers we can make from the prime factors (choose or not choose) which is 2^6.

So there are $2^6 = 64$ possible divisors. However, we must choose at least one prime factor so we can subtract the null set from our possible options. In total there are 63 divisors of 255,255.