Review

Intro to Probability

Page 138 #s 1 - 6

- The probability of an outcome is a measure of how likely it is to occur in a probability experiment.
- The experimental probability of an outcome is based on experimental data. It is defined as the number of favourable outcomes divided by the total number of trials.

$$P(A) = \frac{n(A)}{n(T)}$$

- Subjective probability is an estimate of how likely it is something will occur, based largely on intuition.
- The theoretical probability of an event occurring is a measure of its likelihood based on analysis of all possible outcomes.
- The theoretical probability of an event is calculated by dividing the total number of favourable outcomes by the total number of outcomes in the sample space.
- The probability of the complement of an event is the probability that the event will not occur.
- The odds in favour of an event is the ratio of the probability that the event will happen to the probability that it will not happen.
- The odds against an event occurring is the ratio of the probability that the event will not occur to the probability that it will.

- Probability experiments can be carried out using physical materials or technology-based simulations. Technology-based simulations are useful for carrying out very large numbers of trials.
- Experimental probability approaches theoretical probability as a very large number of trials are conducted.
- Mutually exclusive events cannot occur at the same time.
- To calculate the probability of either mutually exclusive events A or B occurring, use the additive principle (rule of sum) for mutually exclusive events:

  $$P(A \text{ or } B) = P(A) + P(B)$$

- Non-mutually exclusive events can occur at the same time.
- To calculate the number of outcomes included in non-mutually exclusive events A and B, use the principle of inclusion and exclusion:

  $$n(A \text{ or } B) = n(A) + n(B) - n(A \text{ and } B)$$

- To calculate the probability of either non-mutually exclusive events A or B occurring, use the additive principle for non-mutually exclusive events:

  $$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

- Compound events involve more than one event for a given trial of a probability experiment.
- Independent events have no influence on each other's probability of occurring.
- To calculate the probability of two independent events, A and B, both occurring, multiply the probability of each of them occurring:

  $$P(A \text{ and } B) = P(A) \times P(B)$$

- When the occurrence or non-occurrence of one event influences the probability of a second event occurring, the events are dependent.
- To calculate the probability of two dependent events, A and B, both occurring, multiply the probability of the first event occurring by the conditional probability of the second event occurring, given that the first event occurred:

  $$P(A \text{ and } B) = P(A) \times P(B|A)$$

Solutions

1. In a taste-test survey, 140 people out of 210 picked Koala Cola over Brand X. What is the experimental probability that a randomly selected taster will pick
   a) Koala Cola?
   b) Brand X?

a) n(T) = 210, n(A) = 140

$$P(A) = \frac{n(A)}{n(T)}$$

P(A) = 140 / 210

P(A) = 2/3

The experimental probability of choosing Koala Cola is 2/3.

b) n(T) = 210, n(A) = 210 - 140 = 70

$$P(A) = \frac{n(A)}{n(T)}$$

P(A) = 70 / 210

P(A) = 1/3

The experimental probability of choosing Brand X is 1/3.

2. Selia likes to listen to blues, country, and hard rock music. The table shows the songs loaded on her MP3 player.

Type of Song	Number of Songs
Blues	20
Country	30
Hard Rock	50

Selia's player is set to random shuffle, which means that the player randomly plays a song. What are the odds in favour of the player randomly playing

a) a blues song?

b) a hard rock song?

n(S) = 20 + 30 + 50 = 100

a) n(S) = 100, n(A) = 20

$$P(A) = \frac{n(A)}{n(S)}$$

P(A) = 20 / 100

P(A) = 0.2

Therefore P'(A) = 1 - 0.2 = 0.8

Odds in favour are 0.2 : 0.8

(Dividing by 0.2) 1 : 4

The odds in favour of playing a blues song are 1:4.

b) n(S) = 100, n(A) = 50

$$P(A) = \frac{n(A)}{n(S)}$$

P(A) = 50 / 100

P(A) = 0.5

Therefore P'(A) = 1 - 0.5 = 0.5

Odds in favour are 0.5 : 0.5

(Dividing by 0.5) 1 : 1

The odds in favour of playing a hard rock song are 1:1.

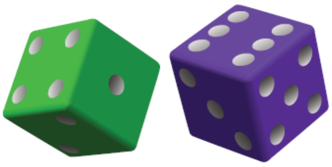
3. **a)** A standard die is rolled 10 times. Explain why it is impossible that the experimental and theoretical probabilities will be perfectly equal.

b) How can this experiment be changed so that they become very close to being equal?

a) The theoretical probability of any of the 6 outcomes on a standard die is 1/6. If we performed 10 trials that would mean we would expect to get 10(1/6) = 1.6666... of each number, which is impossible.

b) The more trials that are performed, the closer the theoretical and experimental probabilities become. So, by significantly increasing the number of trials, you should find that the two sets of anwers become more similar.

4. Determine the probability of rolling a sum of 7 or 11 using a standard pair of dice.



n(S) = 36 (6 x 6)

n(7) = 6 (1,6)(2,5)(3,4)(4,3)(5,2)(6,1)

n(11) = 2 (5,6)(6,5)

P(7 or 11) = P(7) + P(11)

= 6/36 + 2/36

= 8/36

= 2/9

The probability of rolling a sum of 7 or 11 using a pair of standard dice is 2/9.

5. What is the probability of randomly drawing an ace or a red card from a standard deck of playing cards?

n(S) = 52 n(Aces) = 4 n(Red) = 26 n(Red Aces) = 2

P(Ace or Red) = P(Ace) + P(Red) - P(Red Ace)

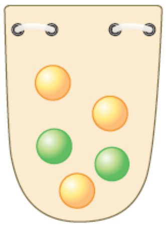
= 4/52 + 26/52 - 2/52

= 28/52

= 7/13

The probability of randomly drawing an ace or a red card from a standard deck is 7/13.

6. Consider the bag of marbles.



a) What is the probability of selecting a yellow marble followed by another yellow marble if the first marble is replaced?

b) How does this answer change if the first marble chosen is not replaced?

c) Explain why these answers are different.

a) As the selected marble is being replaced, these are independent events.

P(YY) = P(Y) x P(Y)

= 3/5 x 3/5

= 9/25

b) As the marble is not replaced, these are dependent events.

P(YY) = P(Y) x P(Y|Y)

= 3/5 x 2/4

= 6/20

= 3/10

c) The answers are different, because the first case does not have replacement, where as the second case does.