Review

1. Topics

- Recursive Sequences
- Arithmetic Sequences
- Geometric Sequences
- Arithmetic Series
- Geometric Series
- Pascal's Triangle
- Binomial Theorem
- Simple Interest
- Compound Interest
- Future Value Annuities
- Present Value Annuities

2. Review Questions

Nelson Page 468 #s 3 (i), 4, 8(ii), 10, 11, 14ad, 15ab, 18ac & 23cd **AND** Nelson Page 534 #s 2, 4, 9, 11, 13 & 15

Mar 19-7:45 AM

Solutions

Nov 20-18:35

3. For each arithmetic sequence, state
i) the general term

a) 58, 73, 88, ...
b) −49, −40, −31, ...
c) 81, 75, 69, ...

a) $a = 58, d = 5$
$\Rightarrow t_n = 58 + 5(n-1)$
$= 58 + 5n - 5$
$= 5n + 53$

b) $a = -49, d = 9$
$\Rightarrow t_n = -49 + 9(n-1)$
$= -49 + 9n - 9$
$= 9n - 58$

c) $a = 81, d = -6$
$\Rightarrow t_n = 81 - 6(n-1)$
$= 81 - 6n + 6$
$= -6n + 87$

May 25-14:14

4. Determine the 100th term of the arithmetic sequence with $t_7 = 465$ and $t_{13} = 219$.

$t_7 = a + d(7-1)$
$\Rightarrow 465 = a + 6d$ ①

$t_{13} = a + d(13-1)$
$\Rightarrow 219 = a + 12d$ ②

Subtracting $\Rightarrow$ ② − ①

$$\begin{aligned} 219 &= a + 12d \\ 465 &= a + 6d \\ \hline -246 &= 6d \end{aligned}$$

$\frac{-246}{6} = \frac{6d}{6}$

$-41 = d$

Using $d = -41$ sub into ①
$\Rightarrow 465 = a + 6(-41)$
$465 = a - 246$
$711 = a$

$\Rightarrow t_n = 711 - 41(n-1)$
$= 711 - 41n + 41$
$= -41n + 752$

May 25-14:16

8. For each geometric sequence, determine

ii) the general term

a) the first term is 7 and the common ratio is −3

b) $a = 12$ and $r = \frac{1}{2}$

c) the second term is 36 and the third term is 144

a) $a = 7$, $r = -3$

$t_n = 7(-3)^{n-1}$

b) $a = 12$, $r = \frac{1}{2}$

$t_n = 12\left(\frac{1}{2}\right)^{n-1}$

c) $r = \frac{144}{36} = 4$

$\frac{36}{a} = 4 \Rightarrow a = \frac{36}{4}$

$a = 9$

$t_n = 9(4)^{n-1}$

May 25-14:16

10. In a laboratory experiment, the count of a certain bacteria doubles every hour.

a) At 1 p.m., there were 23 000 bacteria present. How many bacteria will be present at midnight?

b) Can this model be used to determine the bacterial population at any time? Explain.

a) $a = 23{,}000$, $r = 2$

$n = 12$ [one pm → midnight]

$t_{11} = 23000(2)^{12-1}$

$= 23000(2048)$

$= 47{,}104{,}000$

b) You may think yes, but in reality the bacteria would run out of food and space.

May 25-14:17

11. Guy purchased a rare stamp for \$820 in 2001. If the value of the stamp increases by 10% per year, how much will the stamp be worth in 2010?

$a = 820, \ r = 1.10, \ n = 10 \ [2001 \to 2010]$

$t_{10} = 820(1.10)^{10-1}$

$= 820(1.10)^9$

$= \$1933.52$

May 25-14:18

14. For each arithmetic series, calculate the sum of the first 50 terms.

a) $1 + 9 + 17 + \ldots$

$a = 1, \ d = 8$

$S_{50} = \frac{50}{2}(2(1) + 8(50-1))$

$= 25(2 + 392)$

$= 25(394)$

$= 9850$

d) $-9 - 14 - 19 - \ldots$

$a = -9, \ d = -5$

$S_{50} = \frac{50}{2}(2(-9) - 5(50-1))$

$= 25(-18 - 245)$

$= 25(-263)$

$= -6575$

May 25-14:18

15. Determine the sum of the first 25 terms of an arithmetic series in which

a) the first term is 24 and the common difference is 11

b) $t_1 = 91$ and $t_{25} = 374$

$a = 24,\ d = 11$

$S_{25} = \frac{25}{2}(2(24) + 11(25-1))$

$= 12.5(48 + 264)$

$= 12.5(312)$

$= 3900$

$t_1 = 91 \quad t_{25} = 374$

$S_{25} = \frac{25}{2}(91 + 374)$

$= 12.5(465)$

$= 5812.5$

May 25-14:19

18. For each geometric series, calculate t_6 and S_6.

a) $11 + 33 + 99 + \ldots$

c) $6 - 12 + 24 - \ldots$

$a = 11,\ r = \frac{33}{11} = 3$

$t_6 = 11(3)^{6-1}$

$= 11(3)^5$

$= 2673$

$S_6 = \frac{11(3^6 - 1)}{3 - 1}$

$= \frac{11(728)}{2}$

$= 4004$

$a = 6,\ r = \frac{-12}{6} = -2$

$t_6 = 6(-2)^{6-1}$

$= 6(-2)^5$

$= -192$

$S_6 = \frac{6((-2)^6 - 1)}{-2 - 1}$

$= \frac{6(63)}{-3}$

$= -126$

May 25-14:20

23. Expand and simplify.

c) $(2c+5)^3$

$$= 1(2c)^3(5)^0 + 3(2c)^2(5)^1 + 3(2c)^1(5)^2 + 1(2c)^0(5)^3$$

$$= 8c^3 + 60c^2 + 150c + 250$$

1
1 1
1 2 1
1 3 3 1

d) $(4-3d)^6$ $[1, 6, 15, 20, 15, 6, 1]$

$$= 1(4)^6(-3d)^0 + 6(4)^5(-3d)^1 + 15(4)^4(-3d)^2 + 20(4)^3(-3d)^3 + 15(4)^2(-3d)^4 + 6(4)^1(-3d)^5 + 1(4)^0(-3d)^6$$

$$= 4096 - 18432d + 34560d^2 - 34560d^3 + 19440d^4 - 5832d^5 + 729d^6$$

May 25-14:20

Solutions

Nov 20-18:35

2. Pia invests \$2500 in an account that earns simple interest. At the end of each month, she earns \$11.25 in interest.

a) What annual rate of simple interest is Pia earning? Round your answer to two decimal places.

b) How much money will be in her account after 7 years?

c) How long will it take for her money to double?

a) $I = Prt$

time in years

one month $= \frac{1}{12}$

$11.25 = 2500(r)(\frac{1}{12})$

$\frac{11.25(12)}{2500} = r$

$0.054 = r$

$\Rightarrow$ Interest rate $= 0.054 \times 100\%$ $= 5.40\%$

b) Interest earned $= 7(12)(11.25)$ $= \$945$

$\Rightarrow$ Amount in account $= 2500 + 945$ $= \$3445$

c) For money to double $I = 2500$

$I = Prt$

$2500 = 2500(0.054)(t)$

$\frac{1}{0.054} = \frac{0.054t}{0.054}$

$18.52 = t$

$\Rightarrow$ 18.5 years to double

May 25-14:21

4. Isabelle invests \$4350 at 7.6%/a compounded quarterly. How long will it take for her investment to grow to \$10 000?

Quarterly $= 4$

$P = 4350$

$i = \frac{0.076}{4} = 0.019$

$A = 10000$

$A = P(1+i)^n$

$\frac{10000}{4350} = \frac{4350(1+0.019)^n}{4350}$

$2.29885 = 1.019^n$

using guess and check $\Rightarrow n = 44.2$

Recall: $n = \#\text{years} \times$ "number"

$\Rightarrow \frac{44.2}{4} = \frac{x(4)}{4}$

$11.05 = x$

$\Rightarrow$ 11 years

May 25-14:22

9. Roberto financed a purchase at 9.6%/a compounded monthly for 2.5 years. At the end of the financing period, he still owed $847.53. How much money did Roberto borrow?

Monthly = 12

$i = \frac{0.096}{12} = 0.008$

$n = 2.5(12) = 30$

$A = 847.53$

$P = ?$

$P = A(1+i)^{-n}$

$P = 847.53(1+0.008)^{-30}$

$P = \$667.33$

May 25-14:22

11. For each annuity, calculate the future value and the interest earned.

	Regular Payment	Rate of Compound Interest per Year	Compounding Period	Time
a)	$2500 per year	7.6%	annually	12 years
b)	$500 every 6 months	7.2%	semi-annually	9.5 years
c)	$2500 per quarter	4.3%	quarterly	3 years

$FV = R\left(\frac{(1+i)^n - 1}{i}\right)$

$I = FV - R(n)$

a) $FV = 2500\left(\frac{\left(1+\frac{0.076}{1}\right)^{12\times1} - 1}{\frac{0.076}{1}}\right)$

$FV = 2500\left(\frac{(1.076)^{12} - 1}{0.076}\right)$

$FV = \$46332.35$

$I = 46332.35 - 2500(12)$

$I = \$16332.35$

b) $FV = 500\left(\frac{\left(1+\frac{0.072}{2}\right)^{9.5(2)} - 1}{\frac{0.072}{2}}\right)$

$FV = 500\left(\frac{(1.036)^{19} - 1}{0.036}\right)$

$FV = \$13306.97$

$I = 13306.97 - 500(19)$

$I = \$3806.97$

c) $FV = 2500\left(\frac{\left(1+\frac{0.043}{4}\right)^{3\times4} - 1}{\frac{0.043}{4}}\right)$

$FV = 2500\left(\frac{(1.01075)^{12} - 1}{0.01075}\right)$

$FV = \$31838.87$

$I = 31838.87 - 2500(12)$

$I = \$1838.87$

May 25-14:23

13. Ernie wants to invest some money each month at 9%/a compounded monthly for 6 years. At the end of that time, he would like to have $25 000. How much money does he have to put away each month?

Monthly = 12

FV = 25000

$i = \frac{0.09}{12} = 0.0075$

$n = 6(12) = 72$

$$FV = \frac{R\left((1+i)^n - 1\right)}{i}$$

$$\Rightarrow R = \frac{FV(i)}{\left((1+i)^n - 1\right)}$$

$$R = \frac{25000(0.0075)}{(1.0075)^{72} - 1}$$

$$R = \frac{187.50}{0.71255\ldots}$$

$$R = \$263.14$$

May 25-14:23

15. Paul borrows $136 000. He agrees to make monthly payments for the next 20 years. The interest rate being charged is 6.6%/a compounded monthly.

a) How much will Paul have to pay each month?

b) How much interest is he being charged over the term of the loan?

Monthly = 12

PV = 136000

$i = \frac{0.066}{12} = 0.0055$

$n = 20(12) = 240$

$$PV = \frac{R\left(1-(1+i)^{-n}\right)}{i}$$

$$\Rightarrow R = \frac{PV(i)}{\left(1-(1+i)^{-n}\right)}$$

$$R = \frac{136000(0.0055)}{\left(1-(1.0055)^{-240}\right)}$$

$$R = \frac{748}{0.7318967886}$$

$$R = \$1022.00$$

$$I = R(n) - PV$$

$$I = 1022(240) - 136000$$

$$I = 245280 - 136000$$

$$I = \$109280.00$$

May 25-14:24