Probability Distributions for Continuous Variables

Extra Practice

MHR Page 375 #s 1 - 15

Solutions

1. Which of these variables would be expected to produce a discrete distribution?
   - **A** the distance of a long jump during a track meet
   - **B** the mass of a bolt produced by a factory
   - **C** the number of students with the flu in a given class at your school
   - **D** the total monthly rainfall at a weather station

C

Discrete data is counted - distance, mass and rainfall are all measured.

2. Which of these variables would be expected to produce a continuous distribution?
   - **A** the number of customers at a restaurant at a given time
   - **B** the mass of a hawk recorded during a migration
   - **C** the number of hamburgers sold each day in the school cafeteria
   - **D** the number of defective watches in a shipment to a department store

B

Continuous data is measured - numbers of customers, hamburgers, and defective watches are all counted.

3. The average speeds of five contestants in a bicycle race were 24.2 km/h, 28.1 km/h, 21.6 km/h, 22.0 km/h, and 31.2 km/h. What is the mean of these speeds?

A 22.0 km/h	**B** 24.2 km/h
C 25.4 km/h	**D** 31.2 km/h

C

$$\bar{x} = \frac{\sum x}{n}$$

= (24.2 + 28.1 + 21.6 + 22.0 + 31.2) / 5

= 127.1 / 5

= 25.42

3. The average speeds of five contestants in a bicycle race were 24.2 km/h, 28.1 km/h, 21.6 km/h, 22.0 km/h, and 31.2 km/h.

4. What is the standard deviation of the speeds in #3?

A 3.70 km/h **B** 4.13 km/h

C 4.4 km/h **D** 5.04 km/h

B

Note: we are told the average speeds of 5 of the contestants. This is likely not all of the contestants in the race, so use the sample formula.

$$s = \sqrt{\frac{\Sigma(x-\bar{x})^2}{n-1}}$$

$$= \sqrt{\frac{(24.2-25.4)^2+(28.1-25.4)^2+(21.6-25.4)^2+(22.0-25.4)^2+(31.2-25.4)^2}{5-1}}$$

= 4.13

5. One hundred twenty students qualified for the high jump event at a track meet. The table shows the probability distribution for first jumps.

What is the frequency associated with a jump between 180 cm and 190 cm?

A 0

B 0.117

C 14

D 120

C

Height of Jump (cm)	Probability
120–130	0.025
130–140	0.040
140–150	0.081
150–160	0.150
160–170	0.175
170–180	0.218
180–190	0.117
190–200	0.092
200–210	0.073
210–220	0.017
220–230	0.012

Frequency = n(Probability)

= 120(0.117)

= 14.04

6. Which statement is true concerning a normal distribution?

A The curve may be skewed to the left or right.

B The median is equal to the mean.

C 95% of the data values will occur within one standard deviation of the mean.

D The mean of a sample is always less than the mean of the underlying normal distribution.

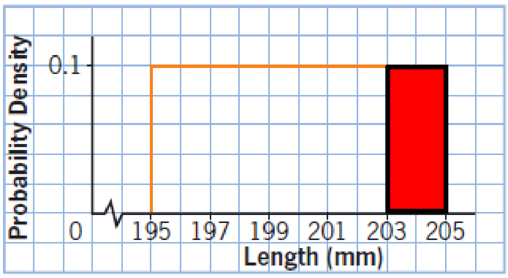
B

A - The curve is not skewed either way.

C - 95% of the data lies within 2 standard deviations, not 1.

D - The mean of a sample can be more (or less) than the mean of the underlying normal distribution.

7. An assembly line produces flip-flops of stated length 200 mm. The graph shows the probability distribution of the actual lengths. What is the probability that a given flip-flop will have a length greater than 203 mm?



Probability is equal to the AREA of the rectangle that gives lengths of greater than 203 mm.

$P(203 < x \leq 205) = 0.1(205 - 203)$

$= 0.1(2)$

$= 0.2$

Probability that a flip-flop will have a length of greater than 203 mm is 0.2

8. Karen's Zumba class has 50 students. She conducts a Zumba endurance contest. The table shows the probability distribution of the participants lasting increasing lengths of time.

Endurance (min)	Probability
30–35	0.04
35–40	0.04
40–45	0.10
45–50	0.20
50–55	0.24
55–60	0.16
60–65	0.10
65–70	0.08
70–75	0.04

a) How many students lasted between 45 min and 50 min?

b) What is the probability that a student lasted less than an hour?

a) Frequency = n(Probability)

= 50(0.2)

= 10

10 students lasted between 45 and 50 minutes.

b) Total the probabilities for less than an hour

= 0.04 + 0.04 + 0.10 + 0.20 + 0.24 + 0.16

= 0.78

Frequency = n(Probability)

= 50(0.78)

= 39

39 students lasted less than an hour.

9. When the ketchup dispenser at a fast-food restaurant is completely depressed, it dispenses a mean of 15 mL of ketchup with a standard deviation of 0.75 mL. Assuming that the distribution is normal, what is the probability that a hamburger will receive less than 14 mL of ketchup in one complete press?

Find the z-score for 14 mL and use the table to find the probability.

$x = 14$, $\overline{x} = 15$, $s = 0.75$

$$z = \frac{x - \overline{x}}{s}$$

z = (14 - 15) / 0.75

z = -1.3333....

⟶ P(x < 14) = 0.0918

The probability that a hamburger will receive less than 14 mL of ketchup in one complete pass is 9.18%

z	0.00	0.01	0.02	0.03
–2.9	0.0019	0.0018	0.0018	0.0017
–2.8	0.0026	0.0025	0.0024	0.0023
–2.7	0.0035	0.0034	0.0033	0.0032
–2.6	0.0047	0.0045	0.0044	0.0043
–2.5	0.0062	0.0060	0.0059	0.0057
–2.4	0.0082	0.0080	0.0078	0.0075
–2.3	0.0107	0.0104	0.0102	0.0099
–2.2	0.0139	0.0136	0.0132	0.0129
–2.1	0.0179	0.0174	0.0170	0.0166
–2.0	0.0228	0.0222	0.0217	0.0212
–1.9	0.0287	0.0281	0.0274	0.0268
–1.8	0.0359	0.0351	0.0344	0.0336
–1.7	0.0446	0.0436	0.0427	0.0418
–1.6	0.0548	0.0537	0.0526	0.0516
–1.5	0.0668	0.0655	0.0643	0.0630
–1.4	0.0808	0.0793	0.0778	0.0764
–1.3	0.0968	0.0951	0.0934	0.0918
–1.2	0.1151	0.1131	0.1112	0.1093

10. Legs for a wooden dining room table are produced by a computer numerically controlled (CNC) lathe. The cutting blade lasts a mean time of 550 h with a standard deviation of 20 h. To avoid cutting errors, the shop manager would like to keep the probability of a failure to less than 0.002. How many hours should the blade be used before replacement? Explain.

Working in reverse this time. Use the table to find the z-score for a probability of 0.002. Then use the z-score formula to find x.

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
–2.9	0.0019	0.0018	0.0018	0.0017	0.0016	0.0016	0.0015	0.0015	0.0014	0.0014
–2.8	~~0.0026~~	~~0.0025~~	~~0.0024~~	~~0.0023~~	~~0.0023~~	~~0.0022~~	~~0.0021~~	~~0.0021~~	0.0020	0.0019
–2.7	0.0035	0.0034	0.0033	0.0032	0.0031	0.0030	0.0029	0.0028	0.0027	0.0026

z = -2.88, $\overline{x}$ = 550, s = 20

$$z = \frac{x - \overline{x}}{s}$$

-2.88 = (x - 550) / 20

-2.88(20) = x - 550

-57.6 = x - 550

492.4 = x

The blade should be replaced after 492.4 hours to keep the probability of failure below 0.002.

11. A pharmaceutical company has determined the probability that a new antacid will relieve stomach distress is 75%. The antacid is given to 1000 patients. The number of patients who reported relief is recorded.

a) Could you reasonably model this distribution using a normal approximation? Explain.

b) Determine the mean and standard deviation of the normal approximation.

a) To model this distribution with a normal distribution both these conditions need to be met: np > 5 and nq > 5.

np = 1000 x 0.75 nq = 1000 x 0.25

np = 750 nq = 250

Both are greater than 5, so it is reasonable to use a normal distribution.

b) $\mu = np$ $\sigma = \sqrt{npq}$

= 1000(0.75) = $\sqrt{1000(0.75)(0.25)}$

= 750 = 13.693...

The mean is 750 and the standard deviation is approximately 13.693

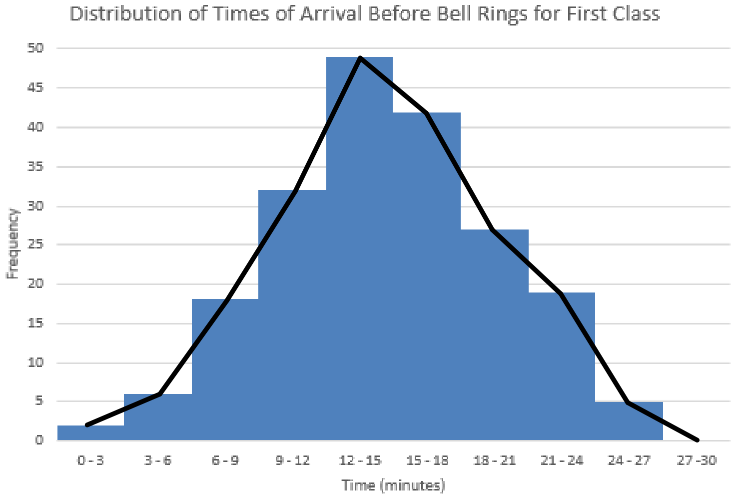
12. Students arrive at school at various times before the bell rings for the first class. The table shows data for 200 students.

a) Sketch a frequency distribution histogram.

b) Add a frequency polygon to the histogram in part a).

c) Do the data appear to follow a normal distribution? Explain your reasoning.

Time (min)	Frequency
0–3	2
3–6	6
6–9	18
9–12	32
12–15	49
15–18	42
18–21	27
21–24	19
24–27	5
27–30	0



c) The data does appear to follow a normal distribution. The central bar is the tallest and the bars drop in height, somewhat symmetrically, to the left and right of the central bar forming a bell-like shape.

13. Airliners taking off from City Central Airport produce a mean noise level of 108 dB (decibels), with a standard deviation of 6.7 dB. To encourage airlines to refit their aircraft with quieter engines, any airliners with a noise level above 120 dB must pay a "nuisance fee."

a) What percent of the airliners will be billed a nuisance fee?

b) After two years, a sample of 1000 airliners showed that 4 were billed the nuisance fee. Assuming that the program has been effective, and that the standard deviation has not changed, what is the new mean noise level?

a) Find the z-score for 120dB and then use the table to find the probability.

x = 120, $\bar{x}$ = 108, s = 6.7

$$z = \frac{x - \bar{x}}{s}$$

z = (120 - 108) / 6.7

z = 1.791... ⟶ P(x < 120) = 0.9633

However, we want P(x > 120)

= 1 - 0.9633

= 0.0367

The percentage of airliners that will be billed a nuisance fee is 3.67%

b) 4 out of 1000 is 0.4%. Working backwards we need to find the z-score for 0.996 (1 - 0.004). Using the table gives a z-score of 2.65. Use the z-score formula to solve for the mean.

z = 2.65, x = 120, s = 6.7

$$z = \frac{x - \bar{x}}{s}$$

2.65 = (120 - $\bar{x}$) / 6.7

2.65(6.7) = 120 - $\bar{x}$

$\bar{x}$ = 120 - 2.65(6.7)

$\bar{x}$ = 102.245 dB

The new mean noise level is about 102 dB.

14. A mail order company is planning to deliver small parcels using remote-controlled drones direct to households within 10 km of its warehouses, each located in a large city. As a test, drones delivered 500 parcels. A total of 420 parcels were delivered within the advertised time limit of 30 min. Determine a 99% confidence interval for the proportion of parcels delivered within 30 min.

Confidence Level	z-Score
90%	1.645
95%	1.960
99%	2.576

A confidence level of 99% has a z-score of 2.576. Use the margin of error formula to find the margin of error. Confidence interval is percentage delivered ± margin of error.

Percentage delivered within 30 minutes

= (420 / 500) x 100%

= 84%

$$E = z\sqrt{\frac{p(1-p)}{n}}$$ where z = 2.576, p = 0.84, n = 500

$$E = 2.576\sqrt{\frac{0.84(1 - 0.84)}{500}}$$

E = 0.0422...

The margin of error at a confidence level of 99% is about 4.2%

Therefore the confidence interval has:

Lower limit = 84 - 4.2

= 79.8%

Upper limit = 84 + 4.2

= 88.2%

The 99% confidence interval for the portion of parcels delivered within 30 minutes is 79.8% to 88.2%

15. A high school operates 450 computers. On a given day, an average of 15 of these computers are out of service. A computer lab contains 30 computers. A teacher would like to use the lab with her class of 27 students.

a) Is this a binomial or a hypergeometric distribution? Explain.

b) Could you reasonably model this distribution using a normal approximation? Explain.

c) Determine the mean and standard deviation of the normal approximation.

d) Use the normal approximation to determine the probability that there will be enough working computers.

a) This is a hypergeometric distribution because there are two outcomes and the outcomes are dependent.

b) Since the number of trials (30) is less than 10% of the population (450) it is reasonable to use a normal distribution to model this scenario.

c) $\mu = np$

= 30(0.967)

= 29.01

$$\sigma = \sqrt{npq\left(\frac{NP - n}{NP - 1}\right)}$$

where p = 435 / 450

p = 0.96666...

$$= \sqrt{30(0.967)(0.033)\frac{450 - 30}{450 - 1}}$$

= 0.9463...

The mean is 29.01 and the standard deviation is about 0.9463

d) For 27 or more successes from 30 computers in the lab, we need to calculate P(26.5 $\leq$ x $\leq$ 30.5).

x = 26.5, $\bar{x}$ = 29.01, s = 0.9463

$$z = \frac{x - \bar{x}}{s}$$

z = (26.5 - 29.01) / 0.9463

z = -2.65... ⟶ P(x < 26.5) = 0.004

x = 30.5, $\bar{x}$ = 29.01, s = 0.9463

$$z = \frac{x - \bar{x}}{s}$$

z = (30.5 - 29.01) / 0.9463

z = 1.57... ⟶ P(x < 30.5) = 0.9418

Therefore P(26.5 < x < 30.5)
= 0.9418 - 0.004
= 0.9378

The probability that 27 or more computers will be working in the lab is 0.9378