Solutions

5. Calculate the future value of each annuity.

	Regular Payment	Rate of Compound Interest per Year	Compounding Period	Time
a)	\$1500 per year	6.3%	annually	10 years
b)	\$250 every 6 months	3.6%	semi-annually	3 years
c)	\$2400 per quarter	4.8%	quarterly	7 years
d)	\$25 per month	8%	monthly	35 years

$$FV = \frac{R\left((1+i)^n - 1\right)}{i}$$

a) $FV = \dfrac{1500\left(\left(1+\frac{0.063}{1}\right)^{10\times1} - 1\right)}{\frac{0.063}{1}}$

$= \dfrac{1500\left((1.063)^{10} - 1\right)}{0.063}$

$= \$20051.96$

b) $FV = \dfrac{250\left(\left(1+\frac{0.036}{2}\right)^{3\times2} - 1\right)}{\frac{0.036}{2}}$

$= \dfrac{250\left((1.018)^{6} - 1\right)}{0.018}$

$= \$1569.14$

c) $FV = \dfrac{2400\left(\left(1+\frac{0.048}{4}\right)^{7\times4} - 1\right)}{\frac{0.048}{4}}$

$= \dfrac{2400\left((1.012)^{28} - 1\right)}{0.012}$

$= \$79308.62$

d) $FV = \dfrac{25\left(\left(1+\frac{0.08}{12}\right)^{35\times12} - 1\right)}{\frac{0.08}{12}}$

$= \dfrac{25\left((1.00\bar{6})^{420} - 1\right)}{0.00\bar{6}}$

$= \$57347.06$

6. Mike wants to invest money every month for 40 years. He would like to have $1 000 000 at the end of the 40 years. For each investment option, how much does he need to invest each month?

a) 10.2%/a compounded monthly

b) 5.1%/a compounded monthly

$$FV = \frac{R((1+i)^n - 1)}{i}$$

a) $$1000000 = \frac{R\left(\left(1+\frac{0.102}{12}\right)^{40\times12} - 1\right)}{\frac{0.102}{12}}$$

$$1000000 = R(6721.69\ldots)$$

$$\Rightarrow R = \$148.77$$

6. Mike wants to invest money every month for 40 years. He would like to have $1 000 000 at the end of the 40 years. For each investment option, how much does he need to invest each month?

a) 10.2%/a compounded monthly

b) 5.1%/a compounded monthly

$$FV = \frac{R(1+i)^n - 1)}{i}$$

$$1{,}000{,}000 = \frac{R\left(1+\frac{0.051}{12}\right)^{40\times12} - 1)}{\frac{0.051}{12}}$$

$$1{,}000{,}000 = R(1566.455\ldots)$$

$$R = \$638.38$$

9. Sonja and Anita want to make equal monthly payments for the next 35 years. At the end of that time, each person would like to have \$500 000. Sonja's bank will give her 6.6%/a compounded monthly. Anita can invest through her work and earn 10.8%/a compounded monthly.
   a) How much more per month does Sonja have to invest?
   b) If Anita decides to invest the same monthly amount as Sonja, how much more money will she have at the end of 35 years?

$$FV = \frac{R\left((1+i)^n - 1\right)}{i}$$

$$\Rightarrow R = FV(i)\left((1+i)^n - 1\right)^{-1}$$

a) Sonja $R = 500000\left(\frac{0.066}{12}\right)\left(\left(1+\frac{0.066}{12}\right)^{35\times 12} - 1\right)^{-1}$

$= 500000(0.0055)\left((1.0055)^{420} - 1\right)^{-1}$

$= \$305.19$

Anita $R = 500000\left(\frac{0.108}{12}\right)\left(\left(1+\frac{0.108}{12}\right)^{35\times 12} - 1\right)^{-1}$

$= 500000(0.009)\left((1.009)^{420} - 1\right)^{-1}$

$= \$106.94$

$\Rightarrow$ Sonja pays $305.19 - 106.94 = \$198.25$ more than Anita per month

b) Anita $FV = \frac{305.19\left((1.009)^{420} - 1\right)}{0.009}$

$= 305.19(42.0814\ldots)(0.009)^{-1}$

$= \$1,426,980.31$

$\Rightarrow$ She will have $1426980.31 - 500000$

$= \$926,980.31$ more than Sonja

Solutions

3. Calculate the present value of each annuity.

	Regular Payment	Rate of Compound Interest per Year	Compounding Period	Time
a)	$5000 per year	7.2%	annually	5 years
b)	$250 every 6 months	4.8%	semi-annually	12 years
c)	$25.50 per week	5.2%	weekly	100 weeks
d)	$48.50 per month	23.4%	monthly	$2\frac{1}{2}$ years

$$PV = \frac{R\left(1-(1+i)^{-n}\right)}{i}$$

a) $$PV = \frac{5000\left(1-\left(1+\frac{0.072}{1}\right)^{-5\times 1}\right)}{\frac{0.072}{1}}$$
$$= \frac{5000(0.29364\ldots)}{0.072}$$
$$= \$20391.67$$

b) $$PV = \frac{250\left(1-\left(1+\frac{0.048}{2}\right)^{-12\times 2}\right)}{\frac{0.048}{2}}$$
$$= \frac{250(0.4340\ldots)}{0.024}$$
$$= \$4521.04$$

c) $$= \frac{25.5\left(1-\left(1+\frac{0.052}{52}\right)^{-100}\right)}{\frac{0.052}{52}}$$
$$= \frac{25.5(0.09511\ldots)}{0.001}$$
$$= \$2425.49$$

d) $$= \frac{48.5\left(1-\left(1+\frac{0.234}{12}\right)^{-2.5(12)}\right)}{\frac{0.234}{12}}$$
$$= \frac{48.5(0.4397\ldots)}{0.0195}$$
$$= \$1093.73$$

4. You want to buy a $1300 stereo on credit and make monthly payments over 2 years. If the store is charging you 18%/a compounded monthly, what will be your monthly payments?

$$PV = \frac{R\left(1-(1+i)^{-n}\right)}{i}$$

$$\Rightarrow R = PV(i)\left(1-(1+i)^{-n}\right)^{-1}$$
$$= 1300\left(\frac{0.18}{12}\right)\left(1-\left(1+\frac{0.18}{12}\right)^{-2\times 12}\right)^{-1}$$
$$= 1300(0.015)(3.3282\ldots)$$
$$= \$64.90$$

7. Emily is investing \$128 000 at 7.8%/a compounded monthly. She wants to withdraw an equal amount from this investment each month for the next 25 years as spending money. What is the most she can take out each month?

$$PV = \frac{R\left(1-(1+i)^{-n}\right)}{i}$$

$$\Rightarrow R = PV(i)\left(1-(1+i)^{-n}\right)^{-1}$$

$$= 128000\left(\frac{0.078}{12}\right)\left(1-\left(1+\frac{0.078}{12}\right)^{-25\times12}\right)^{-1}$$

$$= 128000(0.0065)(1.1670\ldots)$$

$$= \$971.03$$

9. Charles would like to buy a new car that costs \$32 000. The dealership offers to finance the car at 2.4%/a compounded monthly for five years with monthly payments. The dealer will reduce the selling price by \$3000 if Charles pays cash. Charles can get a loan from his bank at 5.4%/a compounded monthly. Which is the best way to buy the car? Justify your answer with calculations.

Dealership

$$R = PV(i)\left(1-(1+i)^{-n}\right)^{-1}$$

$$= 32000\left(\frac{0.024}{12}\right)\left(1-\left(1+\frac{0.024}{12}\right)^{-5\times12}\right)^{-1}$$

$$= 32000(0.002)(8.8516\ldots)$$

$$= \$566.51 \text{ per month}$$

$$\Rightarrow \text{Total paid} = 566.91(60) = \$33990.60$$

Cash via bank loan

$$PV = 32000 - 3000 = 29000$$

$$R = 29000\left(\frac{0.054}{12}\right)\left(1-\left(1+\frac{0.054}{12}\right)^{-5\times12}\right)^{-1}$$

$$= 29000(0.0045)(4.2344\ldots)$$

$$= \$552.60 \text{ per month}$$

$$\Rightarrow \text{Total paid} = 552.60(60) = \$33156.00$$

$\Rightarrow$ Pay cash via bank loan (\$834.60 cheaper)