Solutions

1. A card is randomly drawn from a deck of 52. Drawing a diamond is a success and anything else is a failure. The card is replaced and the deck is shuffled. The experiment is repeated 30 times. To model this experiment using a normal distribution, what mean and standard deviation should you use?

A 7.5, 2.372

B 7.5, 5.625

C 2.739, 2.372

D 2.739, 5.625

A

To approximate the binomial distribution with a normal distribution we have the following:

p = P(Diamond) = 13/52 = 0.25

q = P(No Diamond) = 39/52 = 0.75

n = 30 (number of trials)

$\mu = np$

$= 30(0.25)$

$= 7.5$

$\sigma = \sqrt{npq}$

$= \sqrt{30(0.25)(0.75)}$

$= 2.3717$

2. A bag of jellybeans contains 200 beans, of which 30 are red. Susan reaches into the bag and pulls out 15 beans at random. To model this experiment using a normal distribution, what standard deviation should you use?

A 1.913

B 1.778

C 1.383

D 1.333

D

To approximate the hypergeometric distribution with a normal distribution we have the following:

n = 15 (# of beans selected)

p = 30/200 (# of reds)

q = 170/200 (# of not reds)

NP = 200 (size of population)

$$\sigma = \sqrt{npq\left(\frac{NP-n}{NP-1}\right)}$$

$$= \sqrt{15(30/200)(170/200)\frac{200-15}{200-1}}$$

= 1.333

3. Two dice are rolled. A double is considered a win, and anything else is a loss. What is the minimum number of rolls that should be made to model this situation using a normal distribution?

To be considered a normal distribution np > 5 **AND** nq > 5

6 ways to win (double 1, double 2, ...); 36 possible outcomes (6 x 6)

p = 6/36 q = 30/36

np > 5	nq > 5
n(6/36) > 5	n(30/36) > 5
n > (5 x 36)/6	n > (5 x 36)/30
n > 30	n > 6

For both conditions to be met, the minimum number of rolls is 31.

4. A barrel at the Pro Shop contains 30 white golf balls, 20 yellow golf balls, and 10 orange golf balls. A contest requires a contestant to blindly select several balls without replacement. The prize depends on the number of orange golf balls obtained. What is the maximum number of balls that could be selected to model the contest using a normal approximation?

This is a hypergeometric distribution (golf balls selected without replacement). To model it with a normal distribution the number of dependent trials must be less than 10% of the population.

There are 30 + 20 + 10 = 60 golf balls

10% of 60 = 0.10(60)

= 6

We need to select fewer than 6 golf balls, so the maximum number that could be selected is 5.

5. Five cards are dealt from a deck of 52. The number of hearts is counted.

a) Is it reasonable to model this distribution with a normal distribution? Explain.

b) What mean should you use?

c) What standard deviation should you use?

Dealing cards with no replacement represents a hypergeometric distribution.

a) To model with a normal distribution n < 10% of NP

n = 5 (# of cards dealt) NP = 52 (population)

5 < 0.10(52)

5 < 5.2 which is true.

Yes, it is reasonable to model this distribution with a normal distribution.

b) $\mu = np$

$= 5(0.25)$

$= 1.25$

The mean is 1.25

p = P(Heart) = 13/52 = 0.25

n = 5 (# of cards dealt)

c) $\sigma = \sqrt{npq\left(\frac{NP-n}{NP-1}\right)}$

$= \sqrt{5(0.25)(0.75)\frac{52 - 5}{52 - 1}}$

$= 0.9295$

n = 5 p = 0.25

q = 0.75 NP = 52

The standard deviation is about 0.930

6. A special HOV (high-occupancy vehicle) lane along a highway is reserved for cars carrying two or more people. Police records indicate that 8% of the cars in the HOV lane are occupied by fewer than two people. A random police check observed 100 cars.

a) Use the binomial distribution to determine the probability that exactly 10 of the cars contained one person.
b) Use the normal approximation to determine the probability that exactly 10 of the cars contained one person.
c) Compare the answers to parts a) and b).

n = 100

p = 0.08 (one person)

q = 0.92 (two or more)

a) $P(10) = {}_{100}C_{10}(0.08)^{10}(0.92)^{90}$

$= 0.1024$

The probability that exactly 10 of the cars contained one person is 0.1024

b) We need to apply the continuity correction as we are using a normal approximation. We need to calculate P(9.5 < x < 10.5) so we need the mean, standard deviation and z-scores for 9.5 and 10.5

$\mu = np$

$= 100(0.08)$

$= 8$

$\sigma = \sqrt{npq}$

$= \sqrt{100(0.08)(0.92)}$

$= 2.713$

z	0.00	0.01	0.02	0.03	0.04	0.05
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289

$x = 9.5, \ \bar{x} = 8, \ s = 2.713$

$z = \frac{x - \bar{x}}{s}$

$z = (9.5 - 8) / 2.713$

$z = 0.55 \longrightarrow P(x < 9.5) = 0.7088$

$x = 10.5, \ \bar{x} = 8, \ s = 2.713$

$z = \frac{x - \bar{x}}{s}$

$z = (10.5 - 8) / 2.713$

$z = 0.92 \longrightarrow P(x < 10.5) = 0.8212$

$P(9.5 \leq x \leq 10.5) = 0.8212 - 0.7088$

$= 0.1124$

The probability of exactly 10 successes is 0.1124

c) As np (8) and nq (92) are both > 5, I would expect a close agreement between the two answers.

8. A card is drawn randomly from a deck and then replaced. The deck is shuffled. Ten trials are carried out.

a) Use the binomial distribution to determine the probability that there are exactly 5 diamonds in 10 trials.
b) Could you reasonably model this distribution using a normal approximation? Explain.
c) Determine the mean and standard deviation of the normal approximation.
d) Use the normal approximation to determine the probability of getting exactly 5 diamonds.
e) How does the answer to part d) compare with the answer to part a)?

a) n = 10, p = 0.25, q = 0.75

$P(5) = {}_{10}C_5(0.25)^5(0.75)^5$

$= 0.0584$

The probability that exactly 5 diamonds are drawn is 0.0584

b) Checking the restrictions...

$np = 10(0.25)$

$= 2.5$

$nq = 10(0.75)$

$= 7.5$

Since they are both not > 5 then it is not reasonable to use a normal approximation to model this distribution.

c) $\mu = np$

$= 10(0.25)$

$= 2.5$

$\sigma = \sqrt{npq}$

$= \sqrt{10(0.25)(0.75)}$

$= 1.369$

d) Applying the continuity correction we need to find P(4.5 $\le$ x $\le$ 5.5).
We need the mean, standard deviation, and z-scores for 4.5 and 5.5.

x = 4.5, $\bar{x}$ = 2.5, s = 1.369

$$z = \frac{x - \bar{x}}{s}$$

z = (4.5 - 2.5) / 1.369

z = 1.46 ⟶ P(x < 4.5) = 0.9279

x = 5.5, $\bar{x}$ = 2.5, s = 1.369

$$z = \frac{x - \bar{x}}{s}$$

z = (5.5 - 2.5) / 1.369

z = 2.19 ⟶ P(x < 5.5) = 0.9857

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857

P(4.5 $\le$ x $\le$ 5.5) = 0.9857 - 0.9279
= 0.0578

The probability of exactly 5 diamonds is 0.0578

e) Despite the criteria not being met to use a normal approximation it still gives an answer that is very close, namely 5.8%.

9. Microwave ovens made in China are packaged into containers and shipped to Canada. About 1% are expected to be dented during transport. A sample of five ovens is removed from a container that holds 200. If one is found dented, the container is rejected.

a) Use the hypergeometric distribution to determine the probability that no ovens in the sample are dented.

b) Determine the mean and standard deviation of a normal approximation to this distribution.

c) Use the normal approximation to determine the probability that no ovens in the sample are dented. How does the answer compare with the answer from part a)?

a) n = 200, r = 5, a = 2, Find P(0).

1% of ovens are dented, so we expect 0.01(200) = 2 dented ovens.

$$P(0) = \frac{{}_{2}C_{0} \times {}_{198}C_{5}}{{}_{200}C_{5}}$$

P(0) = 0.9505

The probability that there are no dented ovens in the sample is 0.9505

b) $\mu = np$

= 5(0.01)

= 0.05

$$\sigma = \sqrt{npq\left(\frac{NP - n}{NP - 1}\right)}$$

$$= \sqrt{5(0.01)(0.99)\frac{200 - 5}{200 - 1}}$$

= 0.2202

The mean is 0.05 and the standard deviation is 0.2202

c) Applying the continuity correction we need to find P(-0.5 ≤ x ≤ 0.5).
We need the mean, standard deviation, and z-scores for -0.5 and 0.5.

x = -0.5, $\overline{x}$ = 0.05, s = 0.2202

$$z = \frac{x - \overline{x}}{s}$$

z = (-0.5 - 0.05) / 0.2202

z = -2.50 ⟶ P(x < -0.5) = 0.0202

z	0.00	0.01	0.02	0.03	0.04	0.05
−2.9	0.0019	0.0018	0.0018	0.0017	0.0016	0.0016
−2.8	0.0026	0.0025	0.0024	0.0023	0.0023	0.0022
−2.7	0.0035	0.0034	0.0033	0.0032	0.0031	0.0030
−2.6	0.0047	0.0045	0.0044	0.0043	0.0041	0.0040
−2.5	0.0062	0.0060	0.0059	0.0057	0.0055	0.0054
−2.4	0.0082	0.0080	0.0078	0.0075	0.0073	0.0071
−2.3	0.0107	0.0104	0.0102	0.0099	0.0096	0.0094
−2.2	0.0139	0.0136	0.0132	0.0129	0.0125	0.0122
−2.1	0.0179	0.0174	0.0170	0.0166	0.0162	0.0158
−2.0	0.0228	0.0222	0.0217	0.0212	0.0207	0.0202

x = 0.5, $\overline{x}$ = 0.05, s = 0.2202

$$z = \frac{x - \overline{x}}{s}$$

z = (0.5 - 0.05) / 0.2202

z = 2.04 ⟶ P(x < 0.5) = 0.9793

P(-0.5 ≤ x ≤ 0.5) = 0.9793 - 0.0202
= 0.9591

The probability of exactly 0 dented ovens is 0.9591

z	0.00	0.01	0.02	0.03	0.04
0.0	0.5000	0.5040	0.5080	0.5120	0.5160
0.1	0.5398	0.5438	0.5478	0.5517	0.5557
0.2	0.5793	0.5832	0.5871	0.5910	0.5948
0.3	0.6179	0.6217	0.6255	0.6293	0.6331
0.4	0.6554	0.6591	0.6628	0.6664	0.6700
0.5	0.6915	0.6950	0.6985	0.7019	0.7054
0.6	0.7257	0.7291	0.7324	0.7357	0.7389
0.7	0.7580	0.7611	0.7642	0.7673	0.7704
0.8	0.7881	0.7910	0.7939	0.7967	0.7995
0.9	0.8159	0.8186	0.8212	0.8238	0.8264
1.0	0.8413	0.8438	0.8461	0.8485	0.8508
1.1	0.8643	0.8665	0.8686	0.8708	0.8729
1.2	0.8849	0.8869	0.8888	0.8907	0.8925
1.3	0.9032	0.9049	0.9066	0.9082	0.9099
1.4	0.9192	0.9207	0.9222	0.9236	0.9251
1.5	0.9332	0.9345	0.9357	0.9370	0.9382
1.6	0.9452	0.9463	0.9474	0.9484	0.9495
1.7	0.9554	0.9564	0.9573	0.9582	0.9591
1.8	0.9641	0.9649	0.9656	0.9664	0.9671
1.9	0.9713	0.9719	0.9726	0.9732	0.9738
2.0	0.9772	0.9778	0.9783	0.9788	0.9793

e) The normal approximation gives a slightly higher answer 95.91% versus 95.05% when compared to the hypergeometric distribution.

10. An insurance company knows that 12% of the homeowners in a town of 900 are customers. The marketing department calls 50 homes at random.

a) What is the probability that 10 or more of these are already customers?

b) What method did you choose to solve this problem? Give reasons for your choice.

a) This is a hypergeometric distribution.

n = 50, p = 0.12, q = 0.88, NP = 900

$\mu = np$

= 50(0.12)

= 6

$$\sigma = \sqrt{npq\left(\frac{NP - n}{NP - 1}\right)}$$

$$= \sqrt{50(0.12)(0.88)\frac{900 - 50}{900 - 1}}$$

= 2.2343

x = 9.5, $\overline{x}$ = 6, s = 2.2343

$$z = \frac{x - \overline{x}}{s}$$

z = (9.5 - 6) / 2.2343

z = 1.57 ⟶ P(x < 9.5) = 0.9418

However, we want P(x ≥ 9.5)

= 1 - 09418

= 0.0582

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418

The probability of 10 or more of these already being customers is 0.0582

b) Since the sample size is less than 10% of the population we can use a normal approximation to model the hypergeometric distribution. Besides, it is a lot less calculating than using the indirect method and finding P(0), P(1), P(2), ... P(8), P(9).

11. Application Honey jars from the farm where Doris works say they contain 500 g of honey. A technician measures a sample of 30 jars. The mean content is 502.83 g, with a standard deviation of 1.95 g. The technician can adjust the machine that fills the jars to change the mean. Assume that the standard deviation remains unchanged.

a) Determine the probability that a honey jar contains less than 500 g of honey.

b) Do you need to use a continuity correction factor? Explain.

c) The owners of the company would like to ensure that the probability that a jar contains less than 500 g is at most 0.005. What setting for the mean is required?

a) Find the z-score for 500 g

x = 500, $\overline{x}$ = 502.83, s = 1.95

$$z = \frac{x - \overline{x}}{s}$$

z = (500 - 502.83) / 1.95

z = -1.45 ⟶ P(x < 9.5) = 0.0735

The probability that a jar contains less than 500 g is 0.0735

b) As the mass of honey is continuous data, we don't need to use a continuity correction factor. This is already a normal distribution.

c) We need to work in reverse. To have a probability of 0.005 we need to use the reverse look up in the table to find the appropriate z-score... z = -2.575 (midway between -2.57 and -2.58)

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08
–2.9	0.0019	0.0018	0.0018	0.0017	0.0016	0.0016	0.0015	0.0015	0.0014
–2.8	0.0026	0.0025	0.0024	0.0023	0.0023	0.0022	0.0021	0.0021	0.0020
–2.7	0.0035	0.0034	0.0033	0.0032	0.0031	0.0030	0.0029	0.0028	0.0027
–2.6	0.0047	0.0045	0.0044	0.0043	0.0041	0.0040	0.0039	0.0038	0.0037
–2.5	0.0062	0.0060	0.0059	0.0057	0.0055	0.0054	0.0052	0.0051	0.0049

$$z = \frac{x - \mu}{\sigma}$$

-2.575 = (500 - μ) / 1.95

1.95(-2.575) = 500 - μ

μ = 500 - 1.95(-2.575)

μ = 505.02125

The mean should be set at about 505.02 g to ensure that the probability of a jar being less than 500 g is 0.005