# Solutions

1. Common confidence levels are 90%, 95%, and 99%. A sample is taken from a population with a given mean and standard deviation. Within approximately how many standard deviations of the mean will the values in the confidence interval lie for a

   **a) 90% confidence level?**

   b) 95% confidence level?

   c) 99% confidence level?

| Confidence Level | z-Score |
|---|---|
| 90% | 1.645 |
| 95% | 1.960 |
| 99% | 2.576 |

a) For a 90% confidence level we can expect the data will be within 1.6 standard deviations from the mean.

b) For a 95% confidence level we can expect the data will be within 2 standard deviations from the mean.

c) For a 99% confidence level we can expect the data will be within 2.6 standard deviations from the mean.

2. A poll conducted by the student newspaper found that 78% of the students who ate lunch at the school cafeteria ordered a salad at least twice per week. The poll is considered accurate within ±5%, 17 times out of 20. What is the confidence level for the poll?

A 75%
B 85%
C 95%
D 98%

**B**

Confidence level = 17 / 20
= 0.85

3. During a municipal election the local newspaper polled 251 people. The paper reported that 57% said they were in favour of candidate A for mayor. The result was considered accurate within 6.1%, 19 times out of 20. Which of the following statements is false?

A The margin of error is 6.1%.
B In a similar poll, 95% of the time between 50.9% and 63.1% of the people would be found in favour.
C The confidence interval is 57% ± 6.1%.
D In a similar poll, 95% of the time 57% or more of the people would be found in favour.

**D**

D is false because no range of values is stated

**4.** An automobile dealer offers a new line of tires. The tires last a mean of 75 000 km with a standard deviation of 5000 km, following a normal distribution. The tire life experienced by 100 customers is recorded. What is the expected standard deviation of the sample means?

Check Example 3 on Page 357

$\sigma = 5000$, $n = 100$

$$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$$

$= 5000 / \sqrt{100}$

$= 500$

**The expected standard deviation of the sample means is 500 km.**

**5.** The Canadian commercial pilot written exam consists of 100 multiple choice questions. Last year, the students enrolled in a community college aviation course recorded a mean mark of 82% among 25 candidates.

a) Determine the margin of error at a 99% confidence level.

b) Determine the confidence interval for the exam marks.

c) State the results in the usual format for the course newsletter.

a) $z = 2.576$ (at 99% confidence), $p = 0.82$, $n = 25$

$$E = z\sqrt{\frac{p(1-p)}{n}}$$

$$E = 2.576\sqrt{\frac{0.82(1 - 0.82)}{25}}$$

$E = 0.197933376$

The margin of error at a confidence level of 99% is about 19.8%

b) Confidence interval = Mean ± Margin of Error

Lower limit = 82 - 19.8 = 62.2%

Upper limit = 82 + 19.8 = 101.8%

The 99% confidence interval for these exam marks is 62.2% to 101.8%

c) Usual format is to state the mean, margin of error, and confidence level.

Students recorded a mean mark of 82% ± 19.8%, 99 times out of 100.

**7.** Application A Single Crème cookie is made using a cream filling between two wafers. The amount of cream follows a normal distribution with a mean of 25 g and a standard deviation of 2.0 g. The company claims its new Double Crème line contains twice the amount of filling. A random sample of 20 such cookies were found to contain cream content as shown.

a) Calculate the mean of the sample and the standard deviation for the sample means. What assumption must you make?

b) Determine the 95% confidence interval for the sample mean.

c) Is the company justified in claiming that the Double Crème line contains twice the filling of the Single Crème line? Give reasons for your answer.

| Mass of Cream (g) | | | | |
|---|---|---|---|---|
| 48.9 | 47.3 | 47.3 | 45.5 | 52.9 |
| 50.1 | 46.0 | 47.9 | 48.5 | 48.2 |
| 47.5 | 51.9 | 49.7 | 47.8 | 50.1 |
| 46.9 | 51.0 | 45.9 | 45.4 | 47.1 |

a) Mean = Total mass of the cookies ÷ Number of cookies

= 965.9 ÷ 20

= 48.295 g

$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$

= 2.0 ÷ √20

= 0.44721...

The mean of the sample is 48.3 g with a standard deviation of 0.45 g. We have to assume that the standard deviation is the same for the Double Creme line as it is for the Single Creme line.

b) Confidence interval = Mean ± Margin of Error

Lower limit = 48.3 - 0.882 = 47.418

Upper limit = 48.3 + 0.882 = 49.182

$E = z\sigma_{\bar{x}}$

E = 1.96(0.45)

E = 0.882

The 95% confidence interval for the mean mass of filling is 47.4g to 49.2g

c) No, they are not. We are 95% certain that the range of masses of a Double Creme is between 47.4g and 49.2g. The Single Creme line has a mean of 25g, so at best, they can only make the claim for half of the Double Cremes.

**9.** A honey farm rates its honey on a colour scale from 1 to 20, ranging from very light orange to deep orange. The colour of honey follows a normal distribution with a standard deviation of 2.5. A technician tests a sample of 50 jars of honey, resulting in a sample mean of 12. Determine a 95% confidence interval for the colour of the honey.

Find standard deviation of the mean.

Find the margin of error.

Then find the confidence interval.

$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$

= 2.5 / √50

= 0.353553...

$E = z\sigma_{\bar{x}}$

E = 1.96(0.3536)

E = 0.693

Confidence interval = Mean ± Margin of Error

Lower limit = 12 - 0.693 = 11.307

Upper limit = 12 + 0.693 = 12.693

The 95% confidence interval for the colour of the honey is from about 11 to 13.