

Solutions

Nov 20-18:35

1. The first four entries of the 12th row of Pascal's triangle are 1, 12, 66, and 220.
Determine the first four entries of the 13th row of the triangle.

$$\begin{array}{cccc} 1 & 12 & 66 & 220 \\ = 1 & \swarrow & \swarrow & \swarrow \\ & 13 & 78 & 286 \end{array}$$

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2. Expand and simplify each binomial power.

a) $(x + 2)^5$

$$\begin{aligned}
 &= 1(x)^5(2)^0 + 5(x)^4(2)^1 \\
 &\quad + 10(x)^3(2)^2 + 10(x)^2(2)^3 \\
 &\quad + 5(x)^1(2)^4 + 1(x)^0(2)^5
 \end{aligned}$$

1	1	1		
1	2	1		
1	3	3	1	
1	4	6	4	1
1	5	10	10	5
1	6	15	20	10
1	7	21	35	35
1	8	28	56	56
1	9	36	84	84
1	10	45	120	120
1	11	55	165	165
1	12	66	220	220
1	13	78	285	285
1	14	91	364	364
1	15	105	455	455
1	16	120	560	560
1	17	136	680	680
1	18	153	816	816
1	19	171	951	951
1	20	189	1080	1080
1	21	207	1200	1200
1	22	225	1320	1320
1	23	243	1440	1440
1	24	261	1560	1560
1	25	280	1680	1680
1	26	299	1800	1800
1	27	318	1920	1920
1	28	337	2040	2040
1	29	356	2160	2160
1	30	375	2280	2280
1	31	394	2400	2400
1	32	413	2520	2520
1	33	432	2640	2640
1	34	451	2760	2760
1	35	470	2880	2880
1	36	489	3000	3000
1	37	508	3120	3120
1	38	527	3240	3240
1	39	546	3360	3360
1	40	565	3480	3480
1	41	584	3600	3600
1	42	603	3720	3720
1	43	622	3840	3840
1	44	641	3960	3960
1	45	660	4080	4080
1	46	679	4200	4200
1	47	698	4320	4320
1	48	717	4440	4440
1	49	736	4560	4560
1	50	755	4680	4680
1	51	774	4800	4800
1	52	793	4920	4920
1	53	812	5040	5040
1	54	831	5160	5160
1	55	850	5280	5280
1	56	869	5400	5400
1	57	888	5520	5520
1	58	907	5640	5640
1	59	926	5760	5760
1	60	945	5880	5880
1	61	964	6000	6000
1	62	983	6120	6120
1	63	1002	6240	6240
1	64	1021	6360	6360
1	65	1040	6480	6480
1	66	1059	6600	6600
1	67	1078	6720	6720
1	68	1097	6840	6840
1	69	1116	6960	6960
1	70	1135	7080	7080
1	71	1154	7200	7200
1	72	1173	7320	7320
1	73	1192	7440	7440
1	74	1211	7560	7560
1	75	1230	7680	7680
1	76	1249	7800	7800
1	77	1268	7920	7920
1	78	1287	8040	8040
1	79	1306	8160	8160
1	80	1325	8280	8280
1	81	1344	8400	8400
1	82	1363	8520	8520
1	83	1382	8640	8640
1	84	1401	8760	8760
1	85	1420	8880	8880
1	86	1439	9000	9000
1	87	1458	9120	9120
1	88	1477	9240	9240
1	89	1496	9360	9360
1	90	1515	9480	9480
1	91	1534	9600	9600
1	92	1553	9720	9720
1	93	1572	9840	9840
1	94	1591	9960	9960
1	95	1610	10080	10080
1	96	1629	10200	10200
1	97	1648	10320	10320
1	98	1667	10440	10440
1	99	1686	10560	10560
1	100	1705	10680	10680

$$= x^5 + 10x^4 + 40x^3 + 80x^2 + 80x + 32$$

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4. Expand and simplify each binomial power.

a) $(k + 3)^4$

$$\begin{aligned}
 &= 1(k)^4(3)^0 + 4(k)^3(3)^1 \\
 &\quad + 6(k)^2(3)^2 + 4(k)^1(3)^3 + 1(k)^0(3)^4 \\
 &= k^4 + 12k^3 + 54k^2 + 108k + 81
 \end{aligned}$$

1	1	1		
1	2	1		
1	3	3	1	
1	4	6	4	1
1	5	10	10	5
1	6	15	20	10
1	7	21	35	35
1	8	28	56	56
1	9	36	84	84
1	10	45	120	120
1	11	55	165	165
1	12	66	220	220
1	13	78	285	285
1	14	91	364	364
1	15	105	455	455
1	16	120	560	560
1	17	136	680	680
1	18	153	816	816
1	19	171	951	951
1	20	189	1080	1080
1	21	207	1200	1200
1	22	225	1320	1320
1	23	243	1440	1440
1	24	261	1560	1560
1	25	279	1680	1680
1	26	297	1800	1800
1	27	315	1920	1920
1	28	333	2040	2040
1	29	351	2160	2160
1	30	369	2280	2280
1	31	387	2400	2400
1	32	405	2520	2520
1	33	423	2640	2640
1	34	441	2760	2760
1	35	459	2880	2880
1	36	477	3000	3000
1	37	495	3120	3120
1	38	513	3240	3240
1	39	531	3360	3360
1	40	549	3480	3480
1	41	567	3600	3600
1	42	585	3720	3720
1	43	603	3840	3840
1	44	621	3960	3960
1	45	639	4080	4080
1	46	657	4200	4200
1	47	675	4320	4320
1	48	693	4440	4440
1	49	711	4560	4560
1	50	729	4680	4680
1	51	747	4800	4800
1	52	765	4920	4920
1	53	783	5040	5040
1	54	801	5160	5160
1	55	819	5280	5280
1	56	837	5400	5400
1	57	855	5520	5520
1	58	873	5640	5640
1	59	891	5760	5760
1	60	909	5880	5880
1	61	927	6000	6000
1	62	945	6120	6120
1	63	963	6240	6240
1	64	981	6360	6360
1	65	1000	6480	6480
1	66	1018	6600	6600
1	67	1036	6720	6720
1	68	1054	6840	6840
1	69	1072	6960	6960
1	70	1090	7080	7080
1	71	1108	7200	7200
1	72	1126	7320	7320
1	73	1144	7440	7440
1	74	1162	7560	7560
1	75	1180	7680	7680
1	76	1198	7800	7800
1	77	1216	7920	7920
1	78	1234	8040	8040
1	79	1252	8160	8160
1	80	1270	8280	8280
1	81	1288	8400	8400
1	82	1306	8520	8520
1	83	1324	8640	8640
1	84	1342	8760	8760
1	85	1360	8880	8880
1	86	1378	9000	9000
1	87	1396	9120	9120
1	88	1414	9240	9240
1	89	1432	9360	9360
1	90	1450	9480	9480
1	91	1468	9600	9600
1	92	1486	9720	9720
1	93	1504	9840	9840
1	94	1522	9960	9960
1	95	1540	10080	10080
1	96	1558	10200	10200
1	97	1576	10320	10320
1	98	1594	10440	10440
1	99	1612	10560	10560
1	100	1630	10680	10680

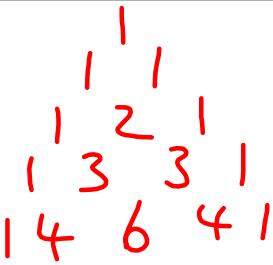
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4. Expand and simplify each binomial power.

K

$$\text{c) } (3q - 4)^4$$

$$\begin{aligned}
 &= 1(3q)^4(-4)^0 + 4(3q)^3(-4)^1 \\
 &\quad + 6(3q)^2(-4)^2 + 4(3q)^1(-4)^3 + 1(3q)^0(-4)^4
 \end{aligned}$$


 A diagram of Pascal's Triangle showing the 4th row. The row consists of six numbers: 1, 4, 6, 4, 1. The first and last numbers are 1. Between them, the numbers 4, 6, and 4 are arranged vertically, with a horizontal line above them and a vertical line to the left of the first 4.

$$= 81q^4 - 432q^3 + 864q^2 - 768q + 256$$

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4. Expand and simplify each binomial power.

K

$$\text{d) } (2x + 7y)^3$$

$$\begin{aligned}
 &= 1(2x)^3(7y)^0 + 3(2x)^2(7y)^1 \\
 &\quad + 3(2x)^1(7y)^2 + 1(2x)^0(7y)^3 \\
 &= 8x^3 + 84x^2y + 294xy^2 + 343y^3
 \end{aligned}$$


 A diagram of Pascal's Triangle showing the 3rd row. The row consists of four numbers: 1, 3, 3, 1. The first and last numbers are 1. Between them, the numbers 3 and 3 are arranged vertically, with a horizontal line above them and a vertical line to the left of the first 3.

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5. Expand and simplify the first three terms of each binomial power.

b) $(3y + 5)^9$

$9^{\text{th}} \text{ row} = 1, 9, 36, 84, 126, 126, 84, 36, 9, 1$

$$\begin{aligned}
 &= 1(3y)^9(5)^0 + 9(3y)^8(5)^1 + 36(3y)^7(5)^2 \\
 &\quad + 84(3y)^6(5)^3 + 126(3y)^5(5)^4 + 126(3y)^4(5)^5 \\
 &\quad + 84(3y)^3(5)^6 + 36(3y)^2(5)^7 + 9(3y)(5)^8 + 1(3y)^0(5)^9 \\
 &= 19683y^9 + 295245y^8 + 1968300y^7 \\
 &\quad + 7654500y^6 + 19136250y^5 + 31893750y^4 \\
 &\quad + 35437500y^3 + 25312500y^2 + 10546875y \\
 &\quad + 1953125
 \end{aligned}$$

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10. Expand and simplify $(3x - 5y)^6$.

$$\begin{aligned}
 &= 1(3x)^6(-5y)^0 + 6(3x)^5(-5y)^1 \quad \begin{array}{ccccccccc} & & & & 1 & 1 & 1 \\ & & & & 1 & 2 & 1 & \\ & & & & 1 & 3 & 3 & 1 \\ & & & & 1 & 4 & 6 & 4 & 1 \\ & & & & 1 & 5 & 10 & 10 & 5 & 1 \\ & & & & 1 & 6 & 15 & 20 & 15 & 6 & 1 \end{array} \\
 &\quad + 15(3x)^4(-5y)^2 + 20(3x)^3(-5y)^3 \\
 &\quad + 15(3x)^2(-5y)^4 + 6(3x)^1(-5y)^5 \\
 &\quad + 1(3x)^0(-5y)^6 \\
 &= 729x^6 - 7290x^5y + 30375x^4y^2 - 67500x^3y^3 \\
 &\quad + 84375x^2y^4 - 56250xy^5 + 15625y^6
 \end{aligned}$$

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