Pascal's Triangle and the Binomial Theorem

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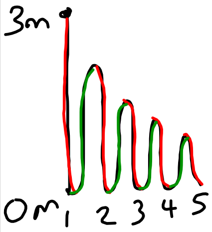
Nov 4-10:28 AM

Warm Up:

A ball is dropped from a height of 3m and bounces on theground. At the top of each bounce, the ball reaches 60%of its previous height. Calculate the total distance travelledby the ball when it hits the ground for the fifth time.First draw a picture!



$S_n = \frac{a(r^n - 1)}{r-1}$

$a = 3,\ r = 0.6,\ n = 5$

$S_5 = \frac{3(0.6^5 - 1)}{0.6 - 1} = 6.9168m$

$a = 1.8$ [60% of 3m], $r = 0.6,\ n = 4$

$S_4 = \frac{1.8(0.6^4 - 1)}{0.6 - 1} = 3.9168m$

Total distance travelled $= 6.9168 + 3.9168$

$= 10.83m$

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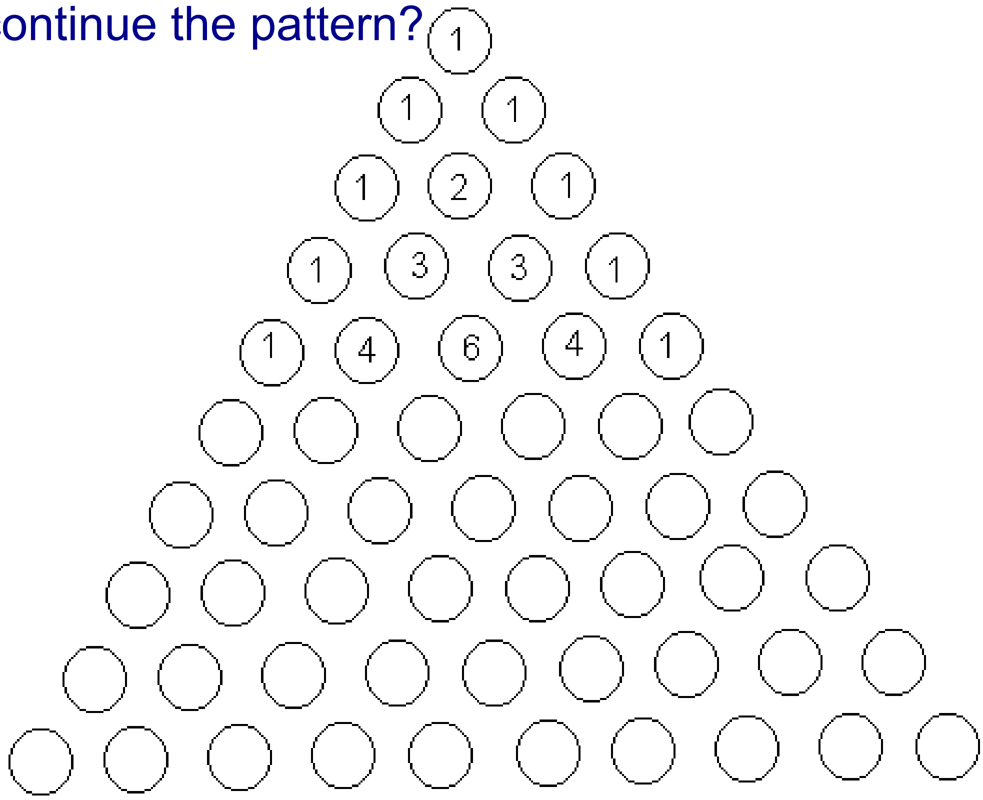
Pascal's Triangle

Can you continue the pattern?



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Pascal's Triangle

A triangular array of numbers where each number in a particular row is equal to the sum of the twonumbers in the row immediately above it.

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1
1 5 10 10 5 1
1 6 15 20 15 6 1
1 7 21 35 35 21 7 1
1 8 28 56 70 56 28 8 1

We number the rows starting at 0

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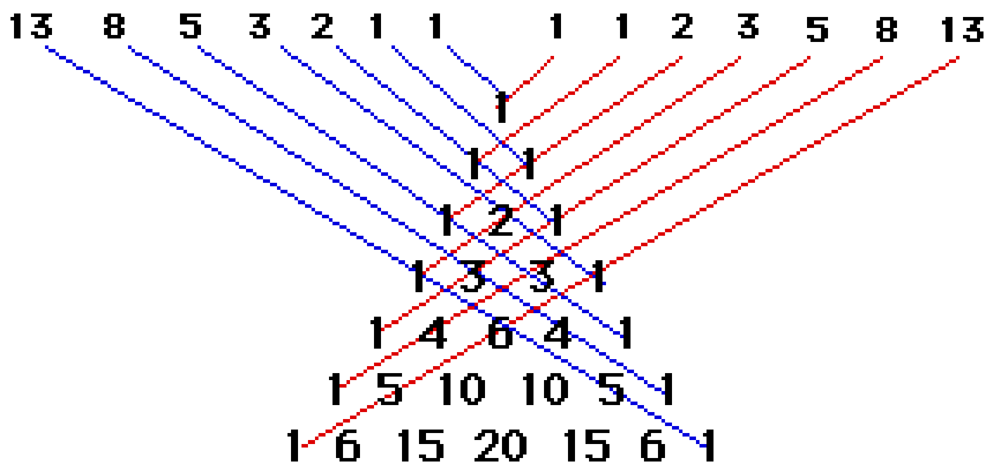
Fibonacci Again!

Powers of 11
$11^0 = 1$
$11^1 = 11$
$11^2 = 121$
$11^3 = 1331$



The Fibonacci Numbers in Pascal's Triangle

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Applying Pascal's Triangle

Expand the following expressions:

$(a+b)^0 = 1$

$(a+b)^1 = a+b$

$(a+b)^2 = a^2+2ab+b^2$

$(a+b)^3 = (a^2+2ab+b^2)(a+b)$

$= a^3+2a^2b+ab^2+a^2b+2ab^2+ab^2+b^3$

$= \mathbf{a^3+3a^2b+3ab^2+b^3}$

$(a+b)^4 = (a^3+3a^2b+3ab^2+b^3)(a+b)$

$= a^4+3a^3b+3a^2b^2+ab^3+a^3b+3a^2b^2+3ab^3+b^4$

$= \mathbf{a^4+4a^3b+6a^2b^2+4ab^3+b^4}$

Pull

Do you notice any patterns with the exponents or coefficients?

For a given exponent, n, the coefficients will be the same as that row number for Pascal's triangle.

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Binomial Theorem

The binomial theorem uses Pascal's Triangle to quickly expand binomials with higher powers.

$$(a+b)^n = c_1a^nb^0 + c_2a^{n-1}b^1 + c_3a^{n-2}b^2 + \ldots + c_{n-1}a^1b^{n-1} + c_na^0b^n$$

where a is the first term in the binomial
b is the second term in the binomial
n is the row of Pascal's triangle

c_1, c_2,..., c_n are the entries in the nth row of Pascal's Triangle

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Example:
Write out the expansion of the following binomials:

a) $(x + 2)^3$ b) $(2x + y)^4$ c) $(3x^2 - 1)^5$

1
1 1
~~1 2 1~~
1 3 3 1
1 4 6 4 1
1 5 10 10 5 1
1 6 15 20 15 6 1
1 7 21 35 35 21 7 1
1 8 28 56 70 56 28 8 1

$= 1(x)^3(2)^0 + 3(x)^2(2)^1 + 3(x)^1(2)^2 + 1(x)^0(2)^3$

$= x^3 + 6x^2 + 12x + 8$

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Example:
Write out the expansion of the following binomials:
a) $(x + 2)^3$ b) $(2x + y)^4$ c) $(3x^2 - 1)^5$

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1
1 5 10 10 5 1
1 6 15 20 15 6 1
1 7 21 35 35 21 7 1
1 8 28 56 70 56 28 8 1

$$= 1(2x)^4(y)^0 + 4(2x)^3(y)^1 + 6(2x)^2(y)^2 + 4(2x)^1(y)^3 + 1(2x)^0(y)^4$$

$$= 16x^4 + 32x^3y + 24x^2y^2 + 8xy^3 + y^4$$

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Example:
Write out the expansion of the following binomials:
a) $(x + 2)^3$ b) $(2x + y)^4$ c) $(3x^2 - 1)^5$

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1
1 5 10 10 5 1
1 6 15 20 15 6 1
1 7 21 35 35 21 7 1
1 8 28 56 70 56 28 8 1

$$= 1(3x^2)^5(-1)^0 + 5(3x^2)^4(-1)^1 + 10(3x^2)^3(-1)^2 + 10(3x^2)^2(-1)^3 + 5(3x^2)^1(-1)^4 + 1(3x^2)^0(-1)^5$$

$$= 243x^{10} - 405x^8 + 270x^6 - 90x^4 + 15x^2 - 1$$

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Homework

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Mar 19-7:45 AM