Solutions

Nov 20-18:35

1. Calculate the sum of the first 10 terms of each arithmetic series.

a) $59 + 64 + 69 + \ldots$

$a = 59, d = 5$

$S_{10} = \frac{10}{2}(2(59) + 5(10-1))$

$S_{10} = 5(118 + 45)$

$S_{10} = 815$

b) $31 + 23 + 15 + \ldots$

$a = 31, d = -8$

$S_{10} = \frac{10}{2}(2(31) + (-8)(10-1))$

$S_{10} = 5(62 + (-72))$

$S_{10} = -50$

c) $-103 - 110 - 117 - \ldots$

$a = -103, d = -7$

$S_{10} = \frac{10}{2}(2(-103) + (-7)(10-1))$

$S_{10} = 5(-206 + (-63))$

$S_{10} = -1345$

d) $-78 - 56 - 34 - \ldots$

$a = -78, d = 22$

$S_{10} = \frac{10}{2}(2(-78) + 22(10-1))$

$S_{10} = 5(-156 + 198)$

$S_{10} = 210$

May 24-15:00

2. Calculate the sum of the first 20 terms of an arithmetic sequence with first term 18 and common difference 11.

$a = 18, d = 11$

$S_{20} = \frac{20}{2}(2(18) + 11(20-1))$

$= 10(36 + 209)$

$= 2450$

May 24-15:00

5. For each series, calculate t_{12} and S_{12}.

K a) $37 + 41 + 45 + 49 + \ldots$ $a = 37, d = 4$

e) $3.19 + 4.31 + 5.43 + 6.55 + \ldots$ $a = 3.19, d = 1.12$

c) $-18 - 12 - 6 + 0 + \ldots$ $a = -18, d = 6$

a) $t_{12} = 37 + 4(12-1)$ $S_{12} = \frac{12}{2}(2(37) + 4(12-1))$

$= 37 + 44$ $= 6(74 + 44)$

$= 81$ $= 708$

e) $t_{12} = 3.19 + 1.12(12-1)$ $S_{12} = \frac{12}{2}(2(3.19) + 1.12(12-1))$

$= 3.19 + 12.32$ $= 6(6.38 + 12.32)$

$= 15.51$ $= 112.2$

c) $t_{12} = -18 + 6(12-1)$ $S_{12} = \frac{12}{2}(2(-18) + 6(12-1))$

$= -18 + 66$ $= 6(-36 + 66)$

$= 48$ $= 180$

May 24-15:00

6. Determine the sum of the first 20 terms of the arithmetic series in which
   a) the first term is 8 and the common difference is 5
   e) the 15th term is 107 and the terms decrease by 3
   c) $t_1 = 53$ and $t_2 = 37$

a) $a = 8, d = 5$

$S_{20} = \frac{20}{2}(2(8) + 5(20-1))$

$= 10(16 + 95)$

$= 1110$

c) $a = 53, d = 37 - 53 = -16$

$S_{20} = \frac{20}{2}(2(53) + (-16)(20-1))$

$= 10(106 - 304)$

$= 10(-198)$

$= -1980$

e) $t_{15} = a + (-3)(15-1)$

$107 = a + (-3)(14)$

$107 = a - 42$

$149 = a$

$S_{20} = \frac{20}{2}(2(149) + (-3)(20-1))$

$= 10(298 - 57)$

$= 10(241)$

$= 2410$

May 24-15:00

7. Calculate the sums of these arithmetic series.
   a) $1 + 6 + 11 + \ldots + 96$
   c) $85 + 77 + 69 + \ldots - 99$

a) $a = 1, d = 5$

$t_n = 1 + 5(n-1)$

$= 1 + 5n - 5$

$= 5n - 4$

$\Rightarrow 96 = 5n - 4$

$\frac{100}{5} = \frac{5n}{5}$

$n = 20$

$\Rightarrow S_{20} = \frac{20}{2}(1 + 96)$

$= 10(97)$

$= 970$

c) $a = 85, d = -8$

$t_n = 85 + (-8)(n-1)$

$= 85 - 8n + 8$

$= -8n + 93$

$\Rightarrow -99 = -8n + 93$

$\frac{-192}{-8} = \frac{-8n}{-8}$

$24 = n$

$\Rightarrow S_{24} = \frac{24}{2}(85 + (-99))$

$= 12(-14)$

$= -168$

May 24-15:00

10. During a skydiving lesson, Chandra jumps out of a plane and falls 4.9 m during the first second. For each second afterward, she continues to fall 9.8 m more than the previous second. After 15 s, she opens her parachute. How far did Chandra fall before she opened her parachute?

$4.9, 14.7, 24.5, \ldots$

$a = 4.9, \; d = 9.8$

$S_{15} = \frac{15}{2}(2(4.9) + 9.8(15-1))$

$= 7.5(9.8 + 137.2)$

$= 7.5(147)$

$= 1102.5 \text{ m}$

May 24-15:00

11. Jamal got a job working on an assembly line in a toy factory. On the 20th day of work, he assembled 137 toys. He noticed that since he started, every day he assembled 3 more toys than the day before. How many toys did Jamal assemble altogether during his first 20 days?

$t_{20} = 137, \; d = 3$

$t_{20} = a + 3(20-1)$

$137 = a + 3(19)$

$137 = a + 57$

$80 = a$

$S_{20} = \frac{20}{2}(80 + 137)$

$= 10(217)$

$= 2170$

May 24-15:00

15. The 10th term of an arithmetic series is 34, and the sum of the first 20 terms is 710. Determine the 25th term.

$t_{10} = 34 = a + d(10-1)$

$34 = a + 9d$ ①

$S_{20} = 710 = \frac{20}{2}(2a + d(20-1))$

$710 = 10(2a + 19d)$

$71 = 2a + 19d$ ②

① $\times 2 \Rightarrow 68 = 2a + 18d$

Subtract ②: $71 = 2a + 19d$

$-3 = -d$

$\Rightarrow d = 3$

Sub $d = 3$ into ① and solve for a

$34 = a + 9(3)$

$34 = a + 27$

$7 = a$

$\Rightarrow t_{25} = 7 + 3(25-1)$

$= 7 + 3(24)$

$= 7 + 72$

$= 79$

May 24-15:00

Solutions

Nov 20-18:35

1. Calculate the sum of the first seven terms of each geometric series.

a) $6 + 18 + 54 + \ldots$

$\times 3$ $\times 3$

$a = 6, r = 3$

$S_7 = \frac{6(3^7 - 1)}{3 - 1}$

$S_7 = 6558$

c) $8 - 24 + 72 - \ldots$

$\times -3$ $\times -3$

$a = 8, r = -3$

$S_7 = \frac{8((-3)^7 - 1)}{(-3) - 1}$

$S_7 = 4376$

b) $100 + 50 + 25 + \ldots$

$\times \frac{1}{2}$ $\times \frac{1}{2}$

$a = 100, r = 0.5$

$S_7 = \frac{100((0.5)^7 - 1)}{0.5 - 1}$

$= 198.4375$

d) $\frac{1}{3} + \frac{1}{6} + \frac{1}{12} + \ldots$

$\times \frac{1}{2}$ $\times \frac{1}{2}$

$a = \frac{1}{3}, r = \frac{1}{2}$

$S_7 = \frac{\frac{1}{3}\left((0.5)^7 - 1\right)}{0.5 - 1}$

$= \frac{\frac{1}{3}\left(\frac{-127}{128}\right)}{-\frac{1}{2}}$

$= \left(\frac{-2}{3}\right)\left(\frac{-127}{128}\right)$

$= \frac{127}{192}$

May 24-15:00

2. Calculate the sum of the first six terms of a geometric sequence with first term 11 and common ratio 4.

$a = 11, r = 4$

$S_6 = \frac{11(4^6 - 1)}{4 - 1}$

$S_6 = \frac{11(4095)}{3}$

$S_6 = 15015$

May 24-15:00

3. For each geometric series, calculate t_6 and S_6.
   a) $6 + 30 + 150 + \ldots$
   e) $3.4 - 7.14 + 14.994 - \ldots$
   c) $21\,000\,000 + 4\,200\,000 + 840\,000 + \ldots$

$a = 6, r = 5$

$a = 3.4, r = -2.1$

$a = 21{,}000{,}000, r = 0.2$

a) $t_6 = 6(5)^{6-1}$

$= 6(5)^5$

$= 18750$

$S_6 = \frac{6(5^6 - 1)}{5 - 1}$

$S_6 = 23436$

e) $t_6 = 3.4(-2.1)^{6-1}$

$= 3.4(-2.1)^5$

$= -138.859434$

$S_6 = \frac{3.4((-2.1)^6 - 1)}{(-2.1) - 1}$

$S_6 = -92.969294$

c) $t_6 = 21{,}000{,}000(0.2)^{6-1}$

$= 21{,}000{,}000(0.2)^5$

$= 6720$

$S_6 = \frac{21{,}000{,}000(0.2^6 - 1)}{0.2 - 1}$

$S_6 = 26{,}248{,}320$

May 24-15:00

5. Determine the sum of the first seven terms of the geometric series in which
   a) $t_1 = 13$ and $r = 5$
   c) $t_1 = 120$ and $t_2 = 30$
   e) $t_8 = 1024$ and the terms decrease by a factor of $\frac{2}{3}$

a) $S_7 = \frac{13(5^7 - 1)}{5 - 1}$

$S_7 = 253{,}903$

c) $a = 120, r = \frac{30}{120} = \frac{1}{4}$

$S_7 = \frac{120\left(\left(\frac{1}{4}\right)^7 - 1\right)}{\frac{1}{4} - 1}$

$S_7 = 159.9902344$

e) $t_8 = a\left(\frac{2}{3}\right)^{8-1}$

$1024 = a\left(\frac{2}{3}\right)^7$

$1024 = a\left(\frac{128}{2187}\right)$

$\Rightarrow a = 1024\left(\frac{2187}{128}\right)$

$a = 17496$

$S_7 = \frac{17496\left(\left(\frac{2}{3}\right)^7 - 1\right)}{\frac{2}{3} - 1}$

$S_7 = 49416$

May 24-15:00

6. Calculate the sum of each geometric series.

a) $1 + 6 + 36 + \ldots + 279\,936$

$a = 1, \quad r = 6$

$t_n = 279936 = (1)(6)^{n-1}$

$279936 = 6^{n-1}$

$6^7 = 6^{n-1}$

$\Rightarrow 7 = n - 1$

$8 = n$

$S_8 = \frac{1(6^8 - 1)}{6 - 1}$

$= 335,923$

c) $17 - 51 + 153 - \ldots - 334\,611$

$a = 17, \quad r = -3$

$t_n = -334611 = 17(-3)^{n-1}$

$\frac{-334611}{17} = \frac{17(-3)^{n-1}}{17}$

$-19683 = (-3)^{n-1}$

$(-3)^9 = (-3)^{n-1}$

$\Rightarrow 9 = n - 1$

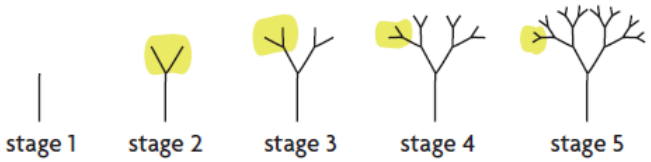
$10 = n$

$S_{10} = \frac{17((-3)^{10} - 1)}{(-3) - 1}$

$= -250,954$

May 24-15:00

9 A simple fractal tree grows in stages. At each new stage, two new line segments branch out from each segment at the top of the tree. The first five stages are shown. How many line segments need to be drawn to create stage 20?



stage 20

$\Rightarrow 1 \quad 2 \quad 4 \quad 8 \quad 16 \quad \ldots\ldots$

$a = 1, \quad r = 2$

$\Rightarrow S_{20} = \frac{1(2^{20} - 1)}{2 - 1}$

$= 1,048,575$

May 24-15:00

11. A large company has a phone tree to contact its employees in case of an emergency factory shutdown. Each of the five senior managers calls three employees, who each call three other employees, and so on. If the tree consists of seven levels, how many employees does the company have?

$a = 5$, $r = 3$, seven levels

$$S_7 = \frac{5(3^7 - 1)}{3 - 1}$$

$$= 5465$$

May 24-15:00