Arithmetic and Geometric Series

Nelson Page 452 #s 1, 2, 5ace, 6ace, 7ac, 10, 11 & 15

AND Page 459 #s 1, 2, 3ace, 5ace, 6ac, 9 & 11

Nov 4-10:28 AM

Warm Up:

Find the general term for the arithmetic sequence that has terms:

$t_{20} = 58$ and $t_{40} = 118$

$t_{20} = a + d(20-1)$

$58 = a + 19d$ ①

$t_{40} = a + d(40-1)$

$118 = a + 39d$ ②

Subtracting ① from ②

$$\Rightarrow \begin{array}{r} 118 = a + 39d \\ 58 = a + 19d \\ \hline 60 = 20d \end{array}$$

$\frac{60}{20} = \frac{20d}{20}$

$3 = d$

Sub $d = 3$ into ①

$58 = a + 19(3)$

$58 = a + 57$

$1 = a$

$\Rightarrow t_n = 1 + 3(n-1)$

$t_n = 1 + 3n - 3$

$t_n = 3n - 2$

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Warm Up 2:

An opera house has 27 seats in the first row,34 seats in the second row, 41 seats in the thirdrow and so on. The last row has 181 seats.

a) How many rows of seats are there in the opera house.

b) How many seats are in the 10th row?

a) $a = 27,\ d = 7,\ t_n = 181$

$t_n = 27 + 7(n-1)$

$181 = 27 + 7n - 7$

$181 = 7n + 20$

$\frac{161}{7} = \frac{7n}{7}$

$23 = n$ $\Rightarrow$ There are 23 rows

b) $t_{10} = 27 + 7(10-1)$

$t_{10} = 27 + 7(9)$

$t_{10} = 27 + 63$

$t_{10} = 90$ seats

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Warm Up 3:

A doctor makes observations of a bacterial culture at fixed time intervals. The table shows his first 4 observations. If the pattern continues, how many bacteria will be present at the 9th observation?

Obs #	# of bacteria
1	5120
2	7680
3	11520
4	17280

$a = 5120,\ r = \frac{7680}{5120} = 1.5$

$t_n = a(r)^{n-1}$

$t_9 = 5120(1.5)^{9-1}$

$t_9 = 5120(1.5)^8$

$t_9 = 131{,}220$ bacteria

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Series

series: is the sum of the terms of a sequence. The sum of the first n terms of a sequence is S_n, where

$$S_n = t_1 + t_2 + t_3 + ... + t_{n-1} + t_n$$

arithmetic series: the sum of an arithmetic sequence

geometric series: the sum of the geometric sequence

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Story Time!

One of the most famous mathematical stories is about a mathematician named Carl Friedrich Gauss. He was a child genius and frequently out-smarted his teachers!

One day his teacher needed some time to get something done and had to occupy her **grade 2** students so she told them to find the sum of the numbers from 1 to 100. (Keep in mind Gauss was in grade 2 in the 1780s.)

Within in 2 minutes Gauss had the answer for his teacher, much to his teacher's disgust!

Take two minutes (without a calculator) and see if you can find the sum of the numbers from 1-100.

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Gauss used a very interesting technique:

He took the sequence of numbers twice and lined then up, so one was forwards and one was backwards:

1 , 2 , 3 , ... , 98, 99, 100
100, 99, 98 , ... , 3 , 2 , 1

Instead of adding horizontally, Gauss added vertically, we can see that each pair will add to 101.

We also know we have 100 pairs so: (100)(101) = 10100
but since we have the sequence twice we need to divide by 2, so 10100/2 = 5050.

If you don't believe him try it at home, add the numbers from 1-100.

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This idea is the basis for the formula for an arithmetic series:

$$S_n = \frac{n}{2}(t_1 + t_n)$$

where S_n = the sum of the series
t_1 is the first term
t_n is the last term
n is the number of terms

we also have another version of the formula ...

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Taking the first formula:

$$S_n = \frac{n}{2}(t_1 + t_n)$$

and the formula for an arithmetic sequence, we know that $t_1 = a$ and $t_n = a + d(n - 1)$ we can sub in these values and then simplify to get another version of the formula. We will need both of these.

$$S_n = \frac{n}{2}(a + a + d(n - 1))$$

$$S_n = \frac{n}{2}(2a + d(n - 1))$$

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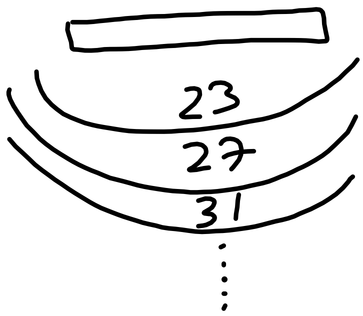
Example:

In an amphitheatre, seats are arranged in 50 semicircular rows facing a domed stage. The first row contains 23 seats, and each row contains 4 more seats than the previous row. How many seats are in the amphitheatre?



$a = 23,\ d = 4,\ n = 50$

$S_n = \frac{n}{2}(2a + d(n-1))$

$S_{50} = \frac{50}{2}(2(23) + 4(50-1))$

$= 25(46 + 196)$

$= 25(242) = 6050$ seats

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Geometric Series

The formula for a geometric series is: $S_n = \dfrac{a(r^n - 1)}{r - 1}$

Example:

At a fish hatchery, fish hatch at different times even though the eggs were all fertilized at the same time. The number of fish that are hatched on each of the first four days was: 2, 10, 50, 250 respectively.

If the pattern continues, calculate the total number of fish hatched in the first 10 days.

$a = 2$, $r = \frac{10}{2} = \frac{50}{10} = \frac{250}{50} = 5$, $n = 10$

$$S_n = \frac{a(r^n - 1)}{r - 1}$$

$$= \frac{2(5^{10} - 1)}{5 - 1} \rightarrow \frac{2(9765625 - 1)}{4}$$

$= 4,882,812$ fish

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Homework

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Mar 19-7:45 AM