Solutions

1. Which of these situations might be reasonably expected to follow a normal distribution?
   - A the heights of students in grade 3 at an elementary school
   - B the mass of peanut butter in a sample of jars marked “1 kg”
   - C the distance that a candidate for the football team can throw a football
   - D all of these

D

Data collected from a large sample of people or naturally occurring events usually have a normal distribution.

2. Which statement is true concerning a normal distribution?

A The curve is symmetrical about a central peak.

B The median is always less than the mean.

C All of the data values will occur within two standard deviations of the mean.

D The mean of a sample always matches the mean of the underlying normal distribution.

A

The others are all false:

B - The median and mean are **equal**

C - 99.7% of the data lie within three standard deviations of the mean

3. A breed of adult cat has a mean mass of 4.2 kg with a standard deviation of 0.5 kg. An article in a pet magazine claims that 1 out of 40 such cats will have a mass of more than 5.2 kg. Does this make sense? Explain.

Use z-scores to work out the probability of a cat having a mass of more than 5.2 kg.

z	0.00	0.01
0.0	0.5000	0.5040
0.1	0.5398	0.5438
0.2	0.5793	0.5832
0.3	0.6179	0.6217
0.4	0.6554	0.6591
0.5	0.6915	0.6950
0.6	0.7257	0.7291
0.7	0.7580	0.7611
0.8	0.7881	0.7910
0.9	0.8159	0.8186
1.0	0.8413	0.8438
1.1	0.8643	0.8665
1.2	0.8849	0.8869
1.3	0.9032	0.9049
1.4	0.9192	0.9207
1.5	0.9332	0.9345
1.6	0.9452	0.9463
1.7	0.9554	0.9564
1.8	0.9641	0.9649
1.9	0.9713	0.9719
2.0	0.9772	0.9778
2.1	0.9821	0.9826

$x = 5.2$, $\overline{x} = 4.2$, $s = 0.5$

$$z = \frac{x - \overline{x}}{s}$$

z = (5.2 - 4.2) / 0.5

z = 2.0 ⟶ P(x < 5.2) = 0.9772

However, we want P(x > 5.2)

= 1 - 0.9772

= 0.0228

There is a 2.3% chance of a cat having a mass of more than 5.2 kg. This represents one cat, and makes sense, because one cat out of forty is 2.5%.

4. Communication A factory produces steel washers that will fit onto standard bolts. Why would a quality control engineer be interested in the mean and standard deviation of these washers?

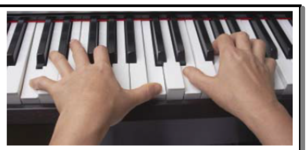
A quality control engineer would be interested in the mean and standard deviation of the washers because it is essential that they will fit a standard sized bolt.

The internal diameter of the washer needs to be bigger than the external diameter of the bolt, otherwise it won't fit.

6. To play an octave on a piano, your hand must span a distance of 16.4 cm. A sample of music students showed a mean hand span of 21.8 cm, with a standard deviation of 2.4 cm.

a) What is the probability that a student could not play an octave?

b) What is the probability that a student could play one and one-half octaves?



Literacy Link

If you start on any white key on a piano, such as C, and count up or down seven white keys, you will reach another C. This interval is known as an *octave*, because it contains eight notes.

a) Find the z-score for a hand span of 16.4 cm

x = 16.4, $\overline{x}$ = 21.8, s = 2.4

$$z = \frac{x - \overline{x}}{s}$$

z = (16.4 - 21.8) / 2.4

z = -2.25 ⟶ P(x < 16.4) = 0.0122

The probability that a student can not play an octave is 0.0122

z	0.00	0.01	0.02	0.03	0.04	0.05
–2.9	0.0019	0.0018	0.0018	0.0017	0.0016	0.0016
–2.8	0.0026	0.0025	0.0024	0.0023	0.0023	0.0022
–2.7	0.0035	0.0034	0.0033	0.0032	0.0031	0.0030
–2.6	0.0047	0.0045	0.0044	0.0043	0.0041	0.0040
–2.5	0.0062	0.0060	0.0059	0.0057	0.0055	0.0054
–2.4	0.0082	0.0080	0.0078	0.0075	0.0073	0.0071
–2.3	0.0107	0.0104	0.0102	0.0099	0.0096	0.0094
–2.2	0.0139	0.0136	0.0132	0.0129	0.0125	0.0122
–2.1	0.0179	0.0174	0.0170	0.0166	0.0162	0.0158

b) To play 1.5 octaves, their span would need to be 1.5(16.4) = 24.6 cm

Again find the z-score, this time for 24.6 cm

x = 24.6, $\overline{x}$ = 21.8, s = 2.4

$$z = \frac{x - \overline{x}}{s}$$

z = (24.6 - 21.8) / 2.4

z = 1.16666.... (1.17)

P(x < 24.6) = 0.8790

z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790

However we want P(x > 24.6)

= 1 - 0.8790

= 0.1210

The probability of a student being able to play one and a half octaves is 0.1210

7. Lithium cells used to power digital watches have a mean lifetime of 8 years with a standard deviation of 1.5 years. The Tempus Watch Company sells a sealed model of watch with power cells that are not replaceable. The watch costs $9.95, and the company offers a 5-year replacement warranty if the cell fails.

a) What percent of its watches would the company expect to replace under a 5-year warranty plan?

b) Assume that the company sells 100 000 watches this year, and it costs $5.00 to replace a defective watch, including shipping. How much should the company budget to replace watches under warranty?

c) An advertising executive suggests boosting sales by offering a 10-year warranty. Is this a reasonable idea? Use calculations similar to parts a) and b) to provide support for your answer.

a) Find the z-score for 5 years

x = 5, $\overline{x}$ = 8, s = 1.5

$$z = \frac{x - \overline{x}}{s}$$

z = (5 - 8) / 1.5

z = -2.0

For a normal distribution 95% of the data lies within 2 standard deviations of the mean. Of the remaining 5% of the data, 2.5% of it would be expected to lie below a z-score of -2 (the other 2.5% would be above a z-score of +2).

b) We can use the z-score to find the probability of a watch failing that has a z-score of -2.0.

⟶ P(x < 5) = 0.0228

Expect replace 0.0228(100,000) = 2280 watches

Cost to replace = 5(2280) = $11,400

They should budget $11,400 to replace the faulty watches

z	0.00	0.01
–2.9	0.0019	0.0018
–2.8	0.0026	0.0025
–2.7	0.0035	0.0034
–2.6	0.0047	0.0045
–2.5	0.0062	0.0060
–2.4	0.0082	0.0080
–2.3	0.0107	0.0104
–2.2	0.0139	0.0136
–2.1	0.0179	0.0174
–2.0	0.0228	0.0222

c) Find the z-score for 10 years

x = 10, $\overline{x}$ = 8, s = 1.5

$$z = \frac{x - \overline{x}}{s}$$

z = (10 - 8) / 1.5

z = 1.3333... (1.33) ⟶ P(x < 10) = 0.9082

z	0.00	0.01	0.02	0.03
0.0	0.5000	0.5040	0.5080	0.5120
0.1	0.5398	0.5438	0.5478	0.5517
0.2	0.5793	0.5832	0.5871	0.5910
0.3	0.6179	0.6217	0.6255	0.6293
0.4	0.6554	0.6591	0.6628	0.6664
0.5	0.6915	0.6950	0.6985	0.7019
0.6	0.7257	0.7291	0.7324	0.7357
0.7	0.7580	0.7611	0.7642	0.7673
0.8	0.7881	0.7910	0.7939	0.7967
0.9	0.8159	0.8186	0.8212	0.8238
1.0	0.8413	0.8438	0.8461	0.8485
1.1	0.8643	0.8665	0.8686	0.8708
1.2	0.8849	0.8869	0.8888	0.8907
1.3	0.9032	0.9049	0.9066	0.9082

This means that 90.82% of the watches are expected to fail within 10 years. Offering a 10 year warranty is not a good idea!

8. The mean mark on a mathematics exam was 70%, with a standard deviation of 10%.

a) If 200 students wrote the exam, how many would be expected to score between 60% and 80%?

b) How many students would be expected to score between 50% and 90%?

c) How many students would be expected to score below 50%?

a) A score of 60% would be one standard deviation below the mean (70 - 10) and a score of 80% would be one standard deviation above the mean (70 + 10). Therefore we would expect 68% of the students to fall into this category, which is 0.68(200) = 136 students.

b) A score of 50% would be two standard deviations below the mean (70 - 10 - 10) and a score of 90% is two standard deviations above the mean (70 + 10 +10). Therefore we would expect 95% of the students to fall into this category, which is 0.95(200) = 190 students.

c) Find the z-score for a score of 50%

x = 50, $\overline{x}$ = 70, s = 10

$$z = \frac{x - \overline{x}}{s}$$

z = (50 - 70) / 10

z = -2.0 ⟶ P(x < 50) = 0.0228

We would expect 0.0228(200) = 4.56 students to score below 50%

z	0.00	0.01
–2.9	0.0019	0.0018
–2.8	0.0026	0.0025
–2.7	0.0035	0.0034
–2.6	0.0047	0.0045
–2.5	0.0062	0.0060
–2.4	0.0082	0.0080
–2.3	0.0107	0.0104
–2.2	0.0139	0.0136
–2.1	0.0179	0.0174
–2.0	0.0228	0.0222