Applications of the Normal Distribution

Lesson objectives

- I can recognise the general characteristics of a normal distribution
- I can use technology to simulate a normal distribution in order to investigate its properties
- I can determine probabilities for a normal distribution

1.1

Lesson objectives | Teachers' notes | Lesson notes

MHR Page 349 #s 1 - 4 & 6 - 8

Warm up

In the fall, apple picking is a popular activity in many orchards across Ontario.

- Would you expect that all of the apples on a tree to have the same mass?
- Are the masses of the apples distributed normally? How can you tell?
- Are all distributions with a central maximum and a bell-like shape normal distributions?

Example

Values Within One Standard Deviation of the Mean

Suppose the birth mass of a breed of guinea pig follows a normal distribution with a mean of 100 g and a standard deviation of 10 g.

a) What is the probability that a birth mass from a large sample lies within one standard deviation of the mean? two standard deviations?

b) Does the answer to part a) depend on the mean and standard deviation of the distribution?

a) The z-scores for lying within 1 standard deviation are -1 and +1

$P(-1 \le z \le 1) = P(z \le 1) - P(z \ge -1)$
$= 0.8413 - 0.1587$
$= 0.6826$

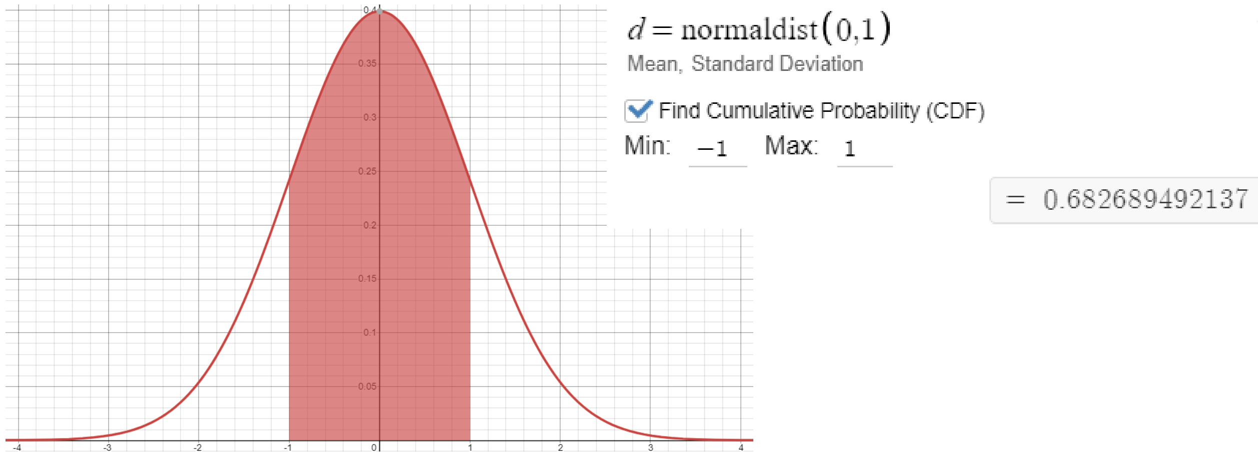
The probability of lying between one standard deviation is about 68%

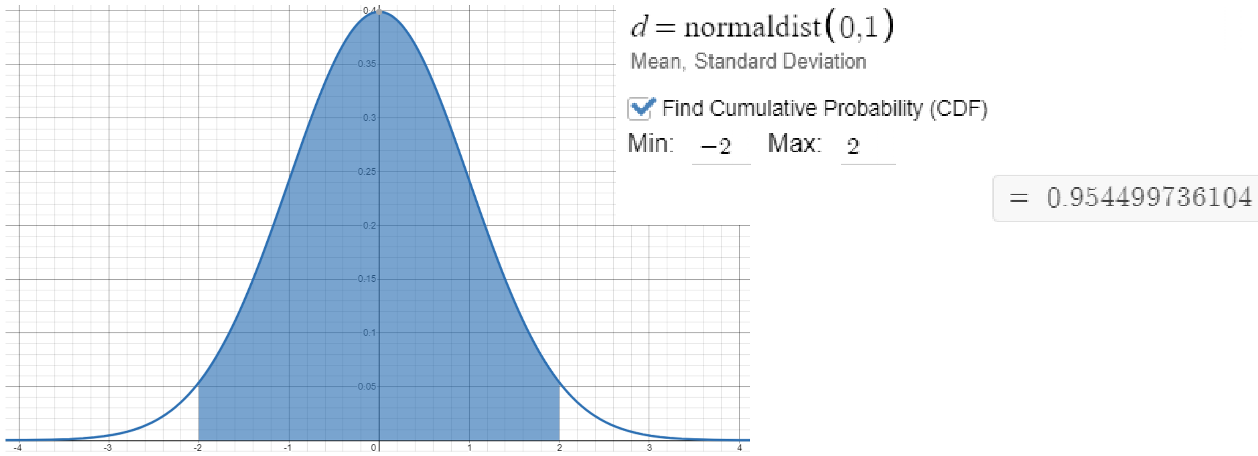
The z-scores for lying within 2 standard deviations are -2 and +2

$P(-2 \le z \le 2) = P(z \le 2) - (z \ge -2)$
$= 0.9772 - 0.0228$
$= 0.9544$

The probability of lying between two standard deviations is about 95%

b) Since one standard deviation from the mean will ALWAYS result in z-scores of ±1 it doesn't matter what the mean and standard deviation are.





Your Turn

A sample of male patients at a hospital showed a mean systolic blood pressure of 124.7 mmHg with a standard deviation of 14.5 mmHg.

a) Use the z-score table on pages 480–481 to determine the probability that a measurement from a large sample lies within three standard deviations of the mean.

b) Use technology to determine the probability that a measurement from a large sample would lie within three standard deviations of the mean.

a) The z-scores for lying within 3 standard deviations are -3 and +3

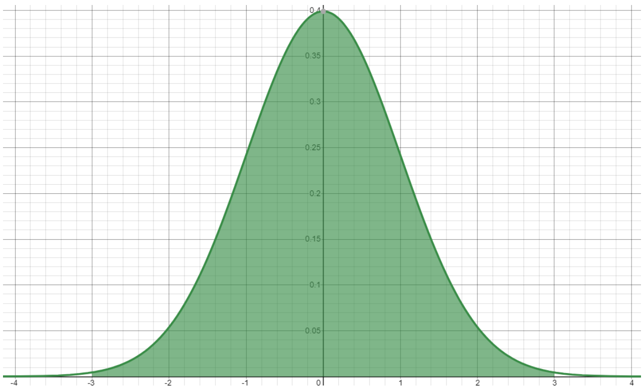
$P(-3 \leq z \leq 3) = P(z \leq 2.99) - P(z \geq -2.99)$

$= 0.9986 - 0.0014$

$= 0.9972$

The probability of lying between three standard deviations is about 99.7%

b)



$d = \text{normaldist}(0,1)$

Mean, Standard Deviation

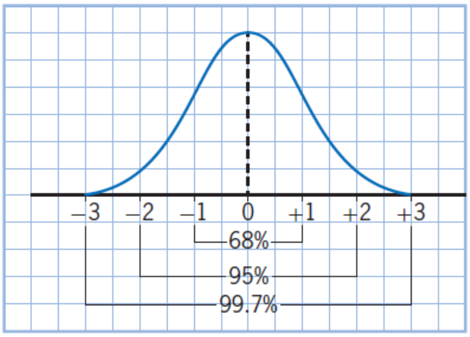
Find Cumulative Probability (CDF)

Min: −3 Max: 3

= 0.997300203937

Key Concepts

- You can use the normal distribution to model the frequency and probability density distributions of continuous random variables.
- The normal distribution has a central peak, and is symmetric about the mean.
- The mean and median are equal.
- About 68% of the data values are within one standard deviation of the mean, about 95% of the data values are within two standard deviations of the mean, and about 99.7% are within three standard deviations of the mean.



R1. A factory produces bolts with a mean length of 5.0 cm and a standard deviation of 0.1 cm. You select a random sample from a batch and find that it has a length of 4.7 cm. Is this a surprising value? Explain your answer.

Determine the z-score for a length of 4.7 cm

Using x = 4.7, $\overline{x}$ = 5, and s = 0.1

$$z = \frac{x - \overline{x}}{s}$$
$$= \frac{4.7 - 5.0}{0.1}$$
$$= -3$$

Yes, this is a surprising value. A z-score of -3 has a probability of only about 0.14%

R2. Machine A fills 1000 one-kilogram honey jars using 1200 kg of honey. Machine B fills 1000 jars using 1050 kg of honey. When the jars are tested, 0.1% of the jars from each machine contain less than 1 kg of honey. Are these results possible? Explain your answer, using diagrams to help you.

Each machine will produce a normal distribution for the mass of honey in a jar. The distributions have different means, one at 1.2 kg and the other at 1.05 kg. Each will produce a "tail" to the left of 1.0 kg. If the standard deviations are correct, the two tail areas will be equal (0.001). Under these conditions, yes, the results are possible.