Solutions

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1. Determine which sequences are geometric. For those that are, state the common ratio.

a) 15, 26, 37, 48, ...

11 11 11

⇒ arithmetic

c) 3, 9, 81, 6561, ...

×3 ×9 ×81

⇒ Something else

b) 5, 15, 45, 135, ...

×3 ×3 ×3

⇒ geometric

Common ratio = 3

d) 6000, 3000, 1500, 750, 375, ...

$\times \frac{1}{2}$ $\times \frac{1}{2}$ $\times \frac{1}{2}$ $\times \frac{1}{2}$

⇒ geometric

Common ratio = 0.5

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3. The 31st term of a geometric sequence is 123 and the 32nd term is 1107. What is the 33rd term?

...... 123, 1107, ?,

$\frac{1107}{123} = 9$ (common ratio)

$\Rightarrow t_{33} = t_{32}(r)$

$= 1107(9)$

$= 9963$

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6. For each geometric sequence, determine

K i) the general term ii) the recursive formula iii) t_6

a) 4, 20, 100, ...

$a = 4 \quad r = 5$

$t_n = ar^{n-1}$

$t_n = 4(5)^{n-1}$

$t_n = 5(t_{n-1})$

where $t_1 = 4$

$t_6 = 4(5)^{6-1}$

$t_6 = 12500$

d) 896, 448, 224, ...

$a = 896 \quad r = 0.5$

$t_n = ar^{n-1}$

$t_n = 896(0.5)^{n-1}$

$t_n = 0.5(t_{n-1})$

where $t_1 = 896$

$t_6 = 896(0.5)^{6-1}$

$t_6 = 28$

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6. For each geometric sequence, determine
K i) the general term ii) the recursive formula iii) t_6

b) $-11, -22, -44, \ldots$

$a = -11 \quad r = 2$

$t_n = ar^{n-1}$

$t_n = -11(2)^{n-1}$

$t_n = 2(t_{n-1})$

where $t_1 = -11$

$t_6 = -11(2)^{6-1}$

$t_6 = -352$

e) $6, 2, \frac{2}{3}, \ldots$

$a = 6 \quad r = \frac{1}{3}$

$t_n = ar^{n-1}$

$t_n = 6\left(\frac{1}{3}\right)^{n-1}$

$t_n = \frac{1}{3}(t_{n-1})$

where $t_1 = 6$

$t_6 = 6\left(\frac{1}{3}\right)^{6-1}$

$t_6 = \frac{2}{81}$

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6. For each geometric sequence, determine
K i) the general term ii) the recursive formula iii) t_6

c) $15, -60, 240, \ldots$

$a = 15 \quad r = -4$

$t_n = ar^{n-1}$

$t_n = 15(-4)^{n-1}$

$t_n = -4(t_{n-1})$

where $t_1 = 15$

$t_6 = 15(-4)^{6-1}$

$t_6 = -15360$

f) $1, 0.2, 0.04, \ldots$

$a = 1 \quad r = \frac{1}{5}$

$t_n = ar^{n-1}$

$t_n = 1\left(\frac{1}{5}\right)^{n-1}$

$t_n = \left(\frac{1}{5}\right)^{n-1}$

$t_n = \frac{1}{5}(t_{n-1})$

where $t_1 = 1$

$t_6 = \left(\frac{1}{5}\right)^{6-1}$

$t_6 = \frac{1}{3125}$

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8. Determine the recursive formula and the general term for the geometric sequence in which

a) the first term is 19 and the common ratio is 5

b) $t_1 = -9$ and $r = -4$

c) the first term is 144 and the second term is 36

a) $a = 19 \quad r = 5$

$t_n = 5(t_{n-1})$

where $t_1 = 19$

$t_n = ar^{n-1}$

$t_n = 19(5)^{n-1}$

b) $a = -9 \quad r = -4$

$t_n = -4(t_{n-1})$

where $t_1 = -9$

$t_n = ar^{n-1}$

$t_n = -9(-4)^{n-1}$

c) $a = 144 \quad r = \frac{36}{144} = \frac{1}{4}$

$t_n = \frac{1}{4}(t_{n-1})$

where $t_1 = 144$

$t_n = ar^{n-1}$

$t_n = 144\left(\frac{1}{4}\right)^{n-1}$

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10. i) Determine whether each general term defines a geometric sequence, where $n \in \mathbb{N}$.

ii) If the sequence is geometric, state the first five terms and the common ratio.

a) $t_n = 4^n$

$t_1 = 4^1 = 4$

$t_2 = 4^2 = 16$

$t_3 = 4^3 = 64$

$t_4 = 4^4 = 256$

$t_5 = 4^5 = 1024$

common ratio $= 4$

d) $t_n = 7 \times (-5)^{n-4}$

$t_1 = 7(-5)^{1-4} = -\frac{7}{125}$

$t_2 = 7(-5)^{2-4} = \frac{7}{25}$

$t_3 = 7(-5)^{3-4} = -\frac{7}{5}$

$t_4 = 7(-5)^{4-4} = 7$

$t_5 = 7(-5)^{5-4} = -35$

common ratio $= -5$

b) $t_n = 3^n + 5$

Not geometric

e) $f(n) = \frac{2}{3n + 1}$

Not geometric

c) $f(n) = n^2 - 13n + 8$

Not geometric

f) $f(n) = \frac{11}{13^n}$

$t_1 = \frac{11}{13^1} = \frac{11}{13}$

$t_2 = \frac{11}{13^2} = \frac{11}{169}$

$t_3 = \frac{11}{13^3} = \frac{11}{2197}$

$t_4 = \frac{11}{13^4} = \frac{11}{28561}$

$t_5 = \frac{11}{13^5} = \frac{11}{371293}$

common ratio $= \frac{1}{13}$

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11. The 5th term of a geometric sequence is 45 and the 8th term is 360. Determine the 20th term.

$t_5 = 45$

$t_n = ar^{n-1}$

$t_5 = ar^4$

$45 = ar^4$ ①

$t_8 = 360$

$t_n = ar^{n-1}$

$t_8 = ar^7$

$360 = ar^7$ ②

Dividing ② by ① gives

$$\frac{360}{45} = \frac{ar^7}{ar^4}$$

$8 = r^3$

$\sqrt[3]{8} = r = 2$

sub $r=2$ into ① or ② gives

$45 = a(2)^4$

$\frac{45}{16} = \frac{16a}{16}$

$2\frac{13}{16} = a$

$\Rightarrow t_{20} = 2\frac{13}{16}(2)^{20-1}$

$= 1474560$

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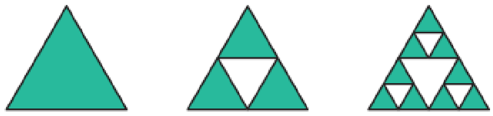
16. The Sierpinski gasket is a fractal created from an equilateral triangle. At each stage, the "middle" is cut out of each remaining equilateral triangle. The first three stages are shown.

a) If the process continues indefinitely, the stages get closer to the Sierpinski gasket. How many shaded triangles would be present in the sixth stage?

b) If the triangle in the first stage has an area of 80 cm^2, what is the area of the shaded portion of the 20th stage?



stage 1 stage 2 stage 3

a) # of shaded Δs $= 1, 3, 9, \ldots$

Look like the powers of $3^{n-1} \Rightarrow 3^{6-1} = 243$

b) Area of stages $= 80, 80\left(\frac{3}{4}\right), 80\left(\frac{3}{4}\right)\left(\frac{3}{4}\right), \ldots$

$\Rightarrow 80\left(\frac{3}{4}\right)^{n-1}$

$t_{20} = 80\left(\frac{3}{4}\right)^{20-1} = 0.338\ \text{cm}^2$

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