Solutions

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1. Determine which sequences are arithmetic. For those that are, state the common difference.

a) 1, 5, 9, 13, 17, ...

+4 +4 +4 +4

Yes. Difference = 4

c) 3, 6, 12, 24, ...

×2 ×2 ×2

No. Difference changes

b) 3, 7, 13, 17, 23, 27, ...

+6 +4 +6 +4 +4

No. Difference is not constant

d) 59, 48, 37, 26, 15, ...

−11 −11 −11 −11

Yes. Difference = −11

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2. State the general term and the recursive formula for each arithmetic sequence.

a) 28, 42, 56, ... b) 53, 49, 45, ... c) −1, −111, −221, ...

a: 14 14, $d = 14$, $a = 28$

b: −4 −4, $d = -4$, $a = 53$

c: −110 −110, $d = -110$, $a = -1$

$t_n = a + d(n-1)$

$t_n = 28 + 14(n-1)$

$t_n = 28 + 14n - 14$

$t_n = 14n + 14$

$t_n = t_{n-1} + 14$ where $t_1 = 28$

$t_n = a + d(n-1)$

$t_n = 53 - 4(n-1)$

$t_n = 53 - 4n + 4$

$t_n = -4n + 57$

$t_n = t_{n-1} - 4$ where $t_1 = 53$

$t_n = a + d(n-1)$

$t_n = -1 - 110(n-1)$

$t_n = -1 - 110n + 110$

$t_n = -110n + 109$

$t_n = t_{n-1} - 110$ where $t_1 = -1$

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4. What is the 15th term of the arithmetic sequence 85, 102, 119, ...?

17 17

$n = 15$

$d = 17$

$a = 85$

$t_n = a + d(n-1)$

$t_{15} = 85 + 17(15-1)$

$t_{15} = 85 + 17(14)$

$t_{15} = 85 + 238$

$t_{15} = 323$

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6. Determine the recursive formula and the general term for the arithmetic sequence in which
   a) the first term is 19 and consecutive terms increase by 8
   b) $t_1 = 4$ and consecutive terms decrease by 5
   c) the first term is 21 and the second term is 26
   d) $t_4 = 35$ and consecutive terms decrease by 12

a) $t_n = t_{n-1} + 8$ where $t_1 = 19$
$t_n = a + d(n-1)$
$t_n = 19 + 8(n-1)$
$t_n = 19 + 8n - 8$
$t_n = 8n + 11$

b) $t_n = t_{n-1} - 5$ where $t_1 = 4$
$t_n = a + d(n-1)$
$t_n = 4 - 5(n-1)$
$t_n = 4 - 5n + 5$
$t_n = -5n + 9$

c) $t_n = t_{n-1} + 5$ where $t_1 = 21$
$t_n = a + d(n-1)$
$t_n = 21 + 5(n-1)$
$t_n = 21 + 5n - 5$
$t_n = 5n + 16$

d) 71, 59, 47, 35, 23
$t_n = t_{n-1} - 12$ where $t_1 = 71$
$t_n = a + d(n-1)$
$t_n = 71 - 12(n-1)$
$t_n = 71 - 12n + 12$
$t_n = -12n + 83$

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7. i) Determine whether each recursive formula defines an arithmetic sequence, where $n \in \mathbf{N}$ and $n > 1$.
   ii) If the sequence is arithmetic, state the first five terms and the common difference.

a) $t_1 = 13, t_n = 14 + t_{n-1}$

13, 27, 41, 55, 69
$\Rightarrow$ arithmetic
common difference = 14

c) $t_1 = 4, t_n = t_{n-1} + n - 1$

4, 5, 7, 10, 14
$\Rightarrow$ not arithmetic

b) $t_1 = 5, t_n = 3t_{n-1}$

5, 15, 45, 135, 405
$\Rightarrow$ not arithmetic

d) $t_1 = 1, t_n = 2t_{n-1} - n + 2$

1, 2, 3, 4, 5
$\Rightarrow$ arithmetic
common difference = 1

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8. For each arithmetic sequence, determine
K i) the general term ii) the recursive formula iii) t_{11}

a) 35, 40, 45, ...
b) 31, 20, 9, ...
c) −29, −41, −53, ...

a) $a=35, d=5$
(i) $t_n = a+d(n-1)$
$t_n = 35+5(n-1)$
$t_n = 35+5n-5$
$t_n = 5n+30$
(ii) $t_n = t_{n-1}+5$ where $t_1 = 35$
(iii) $t_{11} = 5(11)+30$
$t_{11} = 85$

b) $a=31$ $d=-11$
(i) $t_n = 31-11(n-1)$
$t_n = 31-11n+11$
$t_n = -11n+42$
(ii) $t_n = t_{n-1}-11$ where $t_1 = 31$
(iii) $t_{11} = -11(11)+42$
$t_{11} = -79$

c) $a=-29$ $d=-12$
(i) $t_n = -29-12(n-1)$
$t_n = -29-12n+12$
$t_n = -12n-17$
(ii) $t_n = t_{n-1}-12$ where $t_1 = -29$
(iii) $t_{11} = -12(11)-17$
$t_{11} = -149$

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9. i) Determine whether each general term defines an arithmetic sequence.
ii) If the sequence is arithmetic, state the first five terms and the common difference.

a) $t_n = 8-2n$

Arithmetic (linear)

$t_1 = 8-2(1) = 6$
$t_2 = 8-2(2) = 4$
$t_3 = 8-2(3) = 2$
$t_4 = 8-2(4) = 0$
$t_5 = 8-2(5) = -2$

common difference $= -2$

b) $t_n = n^2-3n+7$

Not arithmetic (quadratic)

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10. An opera house has 27 seats in the first row, 34 seats in the second row, 41 seats in the third row, and so on. The last row has 181 seats.

a) How many seats are in the 10th row?

b) How many rows of seats are in the opera house?

a) $a = 27 \quad d = 7$

$t_n = a + d(n-1)$

$t_{10} = 27 + 7(10-1)$

$t_{10} = 27 + 7(9)$

$t_{10} = 27 + 63$

$t_{10} = 90$ seats

b) $t_n = 181$

$t_n = a + d(n-1)$

$\Rightarrow 181 = 27 + 7(n-1)$

$181 = 27 + 7n - 7$

$181 = 7n + 20$

$\frac{161}{7} = \frac{7n}{7}$

$23 = n$

$\Rightarrow$ 23 rows of seats

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13. Determine the number of terms in each arithmetic sequence.

a) 7, 9, 11, 13, ... , 63

$a = 7 \quad d = 2$

$t_n = a + d(n-1)$

$63 = 7 + 2(n-1)$

$\frac{56}{2} = \frac{2(n-1)}{2}$

$28 = n - 1$

$29 = n$

b) −20, −25, −30, −35, ... , −205

$a = -20 \quad d = -5$

$t_n = a + d(n-1)$

$-205 = -20 - 5(n-1)$

$\frac{-185}{-5} = \frac{-5(n-1)}{-5}$

$37 = n - 1$

$38 = n$

c) 31, 27, 23, 19, ... , −25

$a = 31 \quad d = -4$

$t_n = a + d(n-1)$

$-25 = 31 - 4(n-1)$

$\frac{-56}{-4} = \frac{-4(n-1)}{-4}$

$14 = n - 1$

$15 = n$

d) 9, 16, 23, 30, ... , 100

$a = 9 \quad d = 7$

$t_n = a + d(n-1)$

$100 = 9 + 7(n-1)$

$\frac{91}{7} = \frac{7(n-1)}{7}$

$13 = n - 1$

$14 = n$

e) −33, −26, −19, −12, ... , 86

$a = -33 \quad d = 7$

$t_n = a + d(n-1)$

$86 = -33 + 7(n-1)$

$\frac{119}{7} = \frac{7(n-1)}{7}$

$17 = n - 1$

$18 = n$

f) 28, 19, 10, 1, ... , −44

$a = 28 \quad d = -9$

$t_n = a + d(n-1)$

$-44 = 28 - 9(n-1)$

$\frac{-72}{-9} = \frac{-9(n-1)}{-9}$

$8 = n - 1$

$9 = n$

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15. The 50th term of an arithmetic sequence is 238 and the 93rd term is 539. State the general term.

$t_{50} = 238$ $t_{93} = 539$

$t_{50} = a + d(50-1)$ $t_{93} = a + d(93-1)$

$238 = a + 49d$ ① $539 = a + 92d$ ②

Subtracting ① from ②

$539 = a + 92d$

$238 = a + 49d$

$\frac{301}{43} = \frac{43d}{43}$

$7 = d$

use $d = 7$ in ① or ② to solve for a

$238 = a + 49(7)$

$238 = a + 343$

$-105 = a$

$\Rightarrow t_n = a + d(n-1)$

$t_n = -105 + 7(n-1)$

$t_n = -105 + 7n - 7$

$t_n = 7n - 112$

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