Sequences and Patterns

Nov 4-10:26 AM

Warm Up:
Extend the pattern to determine the next three numbers in each sequence:

a) 3, 6, 9, 12, 15, 18, 21, 24 (+3)

b) 5, 6, 8, 11, 15, 20, 26 (increasing by one more than before)

c) 1, 4, 9, 16, 25, 36, 49 (square numbers)

d) -4, 5, 1, 6, 7, 13, 20, 33 (add two previous numbers together)

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Sequences and Patterns

- A sequence is an ordered list of numbers; each number is called a term.

- We number each term based on it's position:

$$t_1, t_2, t_3, t_4, \ldots, t_{n-1}, t_n, \ldots$$

- A finite sequence is a sequence that ends(i.e. 3, 6, 9, 12)

- An infinite sequence is a sequence that continues indefinitely; it ends with ellipsis (...) to show that the pattern continues.

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General Term

The general term of a sequence is the representation of any term in terms of "n", its position number.

i.e. 3, 6, 9, 12, 15 gives $t_n = 3n, n \geq 1$

In other words, the general term is the formula to describe the sequence.

Since the sequence has a starting point we will need to put a restriction on the domain.

$D = \{n \in N\}$ where N is the set of NATURAL numbers (positive integers)

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Example

Determine the general term.

a) 36, 42, 48, 54, ... increases by 6, so linked to 6 times table

6, 12, 18, 24, ...

We need to add 30 to 6x table to get the sequence $\Rightarrow t_n = 6n + 30$

b) 1, 1/4, 1/9, 1/16

Denominators are the square numbers.
Numerator is always one.

$\Rightarrow t_n = \frac{1}{n^2}$

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Sequences and Recursive Formulas

A <u>recursive sequence</u>: a sequence of numbers in which each number is defined by the previous term or terms.

A <u>recursive formula</u>: shows how to find each term from the previous term or terms.

Recursive formulas refer to **at least** one known term.

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Example

Write the first 4 terms.

a) $t_n = 2t_{n-1}$, $t_1 = 1$

$t_2 = 2(t_{2-1})$
$= 2t_1$
$= 2(1)$
$= 2$

$t_3 = 2(t_{3-1})$
$= 2(t_2)$
$= 2(2)$
$= 4$

$t_4 = 2(t_{4-1})$
$= 2t_3$
$= 2(4)$
$= 8$

b) $t_n = 3t_{n-2} - \frac{1}{2}t_{n-1}$, $t_1 = 2$ and $t_2 = 3$

$t_3 = 3t_{3-2} - \frac{1}{2}t_{3-1}$
$= 3t_1 - \frac{1}{2}t_2$
$= 3(2) - \frac{1}{2}(3)$
$= 6 - 1\frac{1}{2}$
$= 4\frac{1}{2}$

$t_4 = 3t_{4-2} - \frac{1}{2}t_{4-1}$
$= 3t_2 - \frac{1}{2}t_3$
$= 3(3) - \frac{1}{2}(4\frac{1}{2})$
$= 9 - 2\frac{1}{4}$
$= 6\frac{3}{4}$

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Example

Write a recursive formula

a) -1, 2, 1, 3, 4, 7,...

$t_n = t_{n-2} + t_{n-1}$

Add the two previous terms together

b) -1, -2, -1, 1, 2, 1, -1,...

$t_n = t_{n-1} - t_{n-2}$

Previous term subtract the one before that

$(-2)-(-1) = -1$
$(-1)-(-2) = 1$
$(1)-(-1) = 2$ etc.

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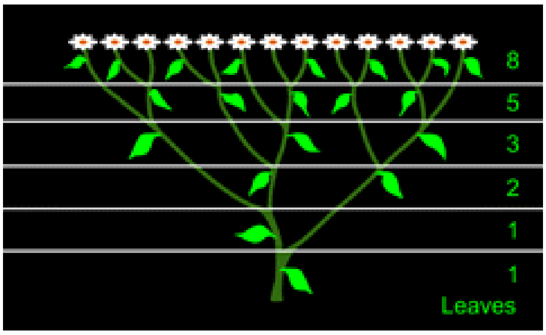
Fibonacci Sequence

A very famous sequence found to be present in various different natural settings

1, 1, 2, 3, 5, 8, 13,...

$t_n = t_{n-2} + t_{n-1}$



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Fibonacci's Bunnies!

The original problem that Fibonacci investigated (in the year 1202) was about how fast rabbits could breed in ideal circumstances.

Suppose a newly-born pair of rabbits, one male, one female, are put in a field. Rabbits are able to mate at the age of one month so that at the end of its second month a female can produce another pair of rabbits.

Now, suppose that our rabbits **never die** and that the female **always** produces one new pair (one male, one female) **every month** from the second month on. The puzzle that Fibonacci posed was... How many pairs will there be in one year?

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Fibonacci's Bunnies!

1. At the end of the first month, they mate, but there is still only one 1 pair.

2. At the end of the second month the female produces a new pair, so nowthere are 2 pairs of rabbits in the field.

3. At the end of the third month, the original female produces a second pair, making 3 pairs in all in the field.

4. At the end of the fourth month, the original female has produced yetanother new pair, the female born two months ago produces her first pairalso, making 5 pairs.

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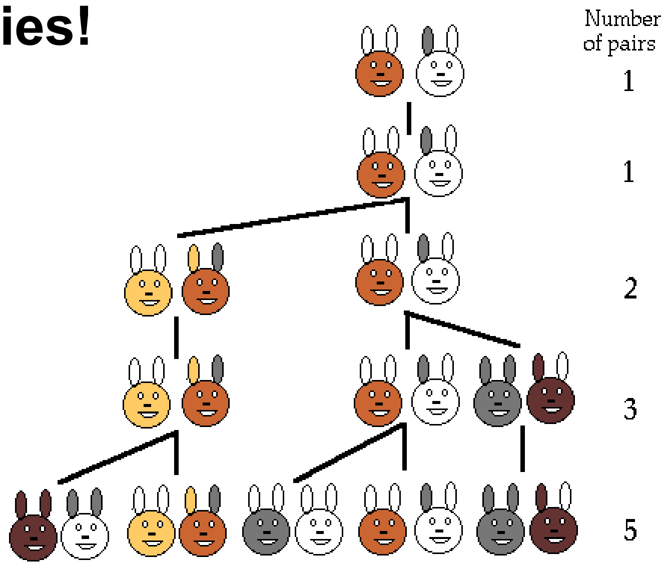
Fibonacci's Bunnies!



http://www.maths.surrey.ac.uk/hosted-sites/R.Knott/Fibonacci/fibnat.html

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Arithmetic Sequences

arithmetic sequence: a linear sequence where the common difference, d, is constant

the general term of an arithmetic sequence:

$$t_n = a + d(n - 1)$$

where

a is the first term

n is the term #

d is the difference

$d = t_n - t_{n-1}$

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Example

Find the formula for the n^{th} term, and then find t_{12}.

a) 8, 12, 16,...

$+4 \quad +4$

$a = 8, d = 4$

$t_n = a + d(n-1)$

$t_n = 8 + 4(n-1)$

$t_n = 8 + 4n - 4$

$t_n = 4n + 4$

b) 1, 6, 11,...

$+5 \quad +5$

$a = 1, d = 5$

$t_n = a + d(n-1)$

$t_n = 1 + 5(n-1)$

$t_n = 1 + 5n - 5$

$t_n = 5n - 4$

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Example

How many terms are in the following sequence?

-3, 2, 7,..., 152

$a = -3, \quad d = 5, \quad t_n = 152$

$t_n = a + d(n-1)$

$\Rightarrow 152 = -3 + 5(n-1)$

$152 = -3 + 5n - 5$

$152 = 5n - 8$

$152 + 8 = 5n$

$\Rightarrow \frac{160}{5} = \frac{5n}{5}$

$32 = n$

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Example

Find the general term given: $t_7 = 121$ and $t_{15} = 193$

$t_7 = a + d(7-1)$ $\quad$ $t_{15} = a + d(15-1)$

$121 = a + 6d$ ① $\quad$ $193 = a + 14d$ ②

Subtracting the linear system ② - ①

$193 = a + 14d$

$121 = a + 6d$

$\frac{72}{8} = \frac{8d}{8}$

$9 = d$

Sub $d = 9$ into ①

$\Rightarrow 121 = a + 6(9)$

$121 = a + 54$

$121 - 54 = a = 67$

$\Rightarrow t_n = 67 + 9(n-1)$

$t_n = 67 + 9n - 9$

$t_n = 9n + 58$

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Homework

Nelson Page 424 #s 1, 2, 4, 6, 7, 8ace, 9ab, 10, 13 & 15

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