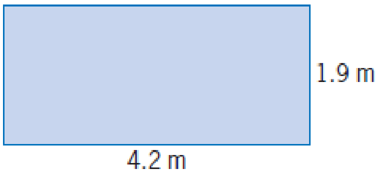
Solutions

1. Determine the area of each figure.

a)

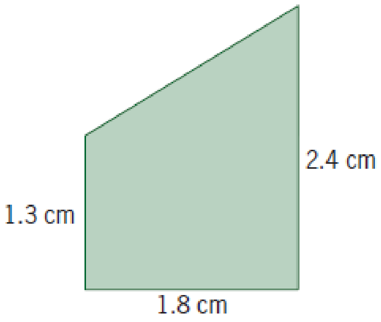


Area = length x width

= 4.2 x 1.9

= 7.98 m^2

b)



Area = $\frac{1}{2}(a+b)h$

= 0.5(1.3 + 2.4)(1.8)

= 3.33 cm^2

2. Eighteen students counted the number of coins in their pockets.

Number of Coins					
3	1	4	3	1	2
4	6	3	5	3	6
5	6	3	2	4	7

a) Sketch a histogram for the data.

b) Sketch a relative frequency histogram for the data.

c) Use technology to determine the mean and standard deviation for the number of coins.

d) Determine the z-score for a student with two coins in her pocket.

A histogram is a graphical display that uses bars of varying heights to represent the frequency with which data occur.

If there are n data points in a data set, then

sample mean: $\bar{x} = \frac{\sum x}{n}$

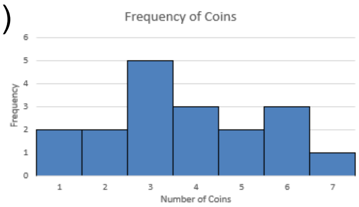
sample standard deviation: $s = \sqrt{\frac{\sum(x-\bar{x})^2}{n-1}}$

The z-score is the number of standard deviations that the value of a continuous variable is from the mean.

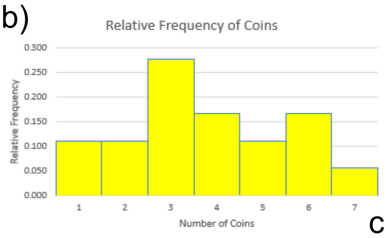
z-score: $z = \frac{x-\bar{x}}{s}$

A data value below the mean has a negative z-score. A data value above the mean has a positive z-score.

a)



b)



Number of Coins	Frequency	Relative Frequency	Freq(x)	Freq(x - mean)^2	
1	2	0.111	2	15.4321	
2	2	0.111	4	6.3210	
3	5	0.278	15	3.0247	
4	3	0.167	12	0.1481	
5	2	0.111	10	2.9877	
6	3	0.167	18	14.8148	
7	1	0.056	7	10.3827	
Total =	18	Mean =	3.7778	1.7675	= Stan Dev

c) Mean = 3.7778, Standard Deviation = 1.7675

d) x = 2, $\bar{x}$ = 3.778, s = 1.7675

$$z = \frac{x-\bar{x}}{s}$$

z = (2 - 3.7778) / 1.7675

z = -1.005827...

The z-score for a student with 2 coins in her pocket is -1.0058.

3. On a final mathematics exam, the students in one class scored a mean of 74 with a standard deviation of 4. The students in another class scored a mean of 72 with a standard deviation of 6.

a) What mark would result in the same z-score for each class?

b) Are there any other marks that would work? Explain why or why not.

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z-score: $z = \frac{x-\bar{x}}{s}$

A data value below the mean has a negative z-score. A data value above the mean has a positive z-score.

a) Class A

$$z = \frac{x-\bar{x}}{s}$$

z = (x - 74) / 4

Class B

$$z = \frac{x-\bar{x}}{s}$$

z = (x - 72) / 6

Solve the linear system

(x - 74) / 4 = (x - 72) / 6

6(x - 74) = 4(x - 72)

6x - 444 = 4x - 288

6x - 4x = -288 + 444

2x = 156

x = 78

A score of 78 would give the same z-score for each class.

b) There are no other marks that would work because there is only one solution to the equation.

4. Thayer collected bonus bills from a hardware store for several months. He found that the 25-cent bill turned up with a relative frequency of 0.4. How many 25-cent bills would be expected in a sample of 80 bills?

$E = np$

$E = 80(0.4)$

$E = 32$

Thayer would expect to get 32 25c bills in his sample of 80.

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5. Evaluate.

- **a)** $5!$
- **b)** ${}_{3}P_{3}$
- **c)** ${}_{10}P_{2}$
- **d)** ${}_{7}C_{4}$

Recall: ${}_{n}P_{r} = n! / (n - r)!$

${}_{n}C_{r} = n! / r!(n - r)!$

a) $5! = 5(4)(3)(2)(1)$

$= 120$

b) ${}_{3}P_{3} = 3! / (3 - 3)!$

$= 3! / 0!$

$= 6$

c) ${}_{10}P_{2} = 10! / (10 - 2)!$

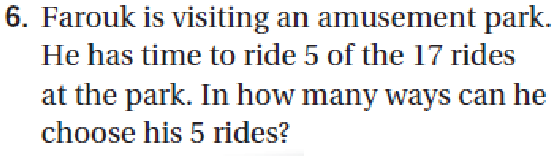
$= 10! / 8!$

$= 90$

d) ${}_{10}P_{2} = 7! / 4!(7 - 3)!$

$= 7! / 4!3!$

$= 35$



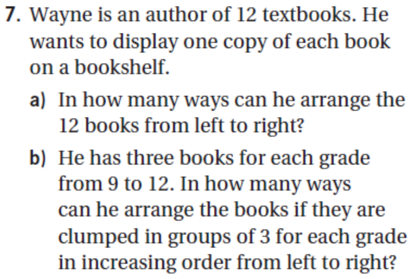
6. Farouk is visiting an amusement park. He has time to ride 5 of the 17 rides at the park. In how many ways can he choose his 5 rides?

It doesn't state that order is important, so we need to find $_{17}C_5$

$= 17! / 5!(17 - 5)!$

$= 6188$

Farouk can choose his five rides in 6,188 different ways.



7. Wayne is an author of 12 textbooks. He wants to display one copy of each book on a bookshelf.

a) In how many ways can he arrange the 12 books from left to right?

b) He has three books for each grade from 9 to 12. In how many ways can he arrange the books if they are clumped in groups of 3 for each grade in increasing order from left to right?

a) Order is important, so he can arrange them in 12! ways.

Or... $_{12}P_{12} = 12! / (12 - 0)!$

$= 479,001,600$ ways

b) Arrange the G9s, then the G10s, then G11s and then G12s

$= {}_3P_3 \times {}_3P_3 \times {}_3P_3 \times {}_3P_3$ Or... $3! \times 3! \times 3! \times 3!$

$= 6 \times 6 \times 6 \times 6$

$= 1296$ ways

8. Sam purchases 5 green gumballs, 7 red gumballs, and 3 white gumballs. The gumballs are in a paper bag. He reaches into the bag and pulls out one gumball. What is the probability that it is green?

Total of 5 + 7 + 3 = 15 gumballs

P(Green) = 5 / 15

= 1/3

Probability of selecting a green gumball is 1/3.

9. Six students are lined up to purchase tickets for a movie. What is the probability that they are lined up in alphabetical order by first name?

There are 6! (or 6P6) ways of the students lining up.

Only one way will be in alphabetical order by first name.

P(alphabetical order) = 1 / 6!

= 1/720

10. The Drama Club holds a draw at each performance to raise money for props and costumes. They sell 200 tickets at $2 each. There is one prize of each of $100, $75, and $25. What is the expected value of each ticket sold?

$$E(X) = \frac{\text{Total value of prizes}}{\text{Number of tickets sold}} - \text{price of ticket}$$

$$= \frac{100 + 75 + 25}{200} - 2$$

$$= 1 - 2$$

$$= -\$1$$

The expected value of each ticket is -$1.00

11. A board game uses a 12-sided die as shown. It is rolled 7 times. What is the probability that it comes up greater than 9 exactly 4 times?

These are independent trials with options of success or failure.

This is a binomial distribution.

To get greater than a 9, you need to roll a 10, 11, or 12.

P(Greater than 9) = 3/12 = 0.25

$p = 0.25, q = 0.75, n = 7, x = 4$

\# of arrangements of the successes

Successes

$$P(\text{exactly 4 rolls greater than 9}) = {}_7C_4(0.25)^4(0.75)^3$$

$$= 0.057678...$$

Failures

The probability of rolling exactly 4 rolls greater than 9 on a 12-sided die is about 0.0577.

12. The barriers at a commuter train crossing are down for a total of 12 min every hour. If 100 cars approach the barrier every hour, what is the probability that exactly 20 will find the barriers down?

P(Barrier down) = 12 / 60

= 0.2

$p = 0.2$, $q = 0.8$, $n = 100$, $x = 20$

$$P(\text{Exactly } 20) = {}_{100}C_{20}(0.2)^{20}(0.8)^{80}$$

$$= 0.0993...$$

The probability of the barrier is down for exactly 20 cars out of the 100 is about 0.0993.

13. A mathematics class has 4 students with red hair, 6 with black hair, 9 with brown hair, and 7 with blond hair. The teacher randomly chooses 4 students to plan a class pizza party. What is the probability that all 4 have red hair?

Total of 4 + 6 + 9 + 7 = 26 students.

There are ${}_{26}C_4 = 14{,}950$ ways to select 4 students.

$$P(4 \text{ red hair}) = \frac{{}_4C_4 \times {}_{22}C_0}{{}_{26}C_4}$$

$$= \frac{1}{14950}$$

$$= 6.689 \times 10^{-5}$$

The probability of all four having red hair is about 0.0067%.

14. Of the 1000 students at Eastdale Secondary School, 420 are boys. For a promotion, the cafeteria manager selects 10 students at random to receive a free sample of a new wrap. Use technology to determine the probability that an equal number of boys and girls will receive a free sample.

10 students are chosen, so to have an equal number of boys and girls we need to select 5 of each.

There 420 boys, so there are 1000 - 420 = 580 girls to choose from.

$$P(\text{5 Boys AND 5 Girls}) = \frac{{}_{420}C_5 \times {}_{580}C_5}{{}_{1000}C_{10}}$$

$$= 0.2170...$$

The probability of choosing an equal number of boys and girls to receive a free sample is about 21.7%.

15. A department store mails out Saturday-Scratch-n-Save cards to all of the households in a large city. One card in 100 offers a discount of 50%, while the rest offer 5%. Last Saturday, 250 customers used their cards.

a) What is the probability that exactly 3 customers received a 50% discount?

b) What is the probability that more than 1 but fewer than 4 customers received a 50% discount?

$P(\text{50\% discount}) = 1/100 = 0.01$

a) $p = 0.01, \quad q = 0.99, \quad n = 250, \quad x = 3$

$$P(\text{Exactly 3 50\% discounts}) = {}_{250}C_3(0.01)^3(0.99)^{247}$$

$$= 0.2149...$$

The probability of exactly three 50% discount cards being used from 250 customers is about 21.49%.

b) $p = 0.01, \quad q = 0.99, \quad n = 250, \quad x = 2 \text{ or } 3$

$$P(\text{Exactly 2 50\% discounts}) = {}_{250}C_2(0.01)^2(0.99)^{248}$$

$$= 0.2574...$$

$$\text{Probability}(1 < x < 4) = P(2) + P(3)$$

$$= 0.2574 + 0.2149$$

$$= 0.4723$$

The probability of more than one, but less than four 50% discount cards being used is 47.23%.