Review

1. Review Sheet Topics:
   - Exponent Laws
   - Rational Exponents
   - Graphs of Exponential Functions
   - Properties of Exponential Functions
   - Transformations
   - Solving Exponential Functions
   - Growth and Decay Problems

2. Review Questions
   Nelson Page 267 #s 2 - 5, 7 & 9 - 17

Mar 19-7:45 AM

Solutions

Nov 20-18:35

2. Write each as a single power. Then evaluate. Express answers in rational form.

a) $(-7)^3(-7)^{-4}$ $= (-7)^{3+(-4)} = (-7)^{-1} = -\frac{1}{7}$

b) $\frac{(-2)^8}{(-2)^3}$ $= (-2)^{8-3} = (-2)^5 = -32$

c) $\frac{(5)^{-3}(5)^6}{5^3}$ $= (5)^{-3+6-3} = 5^0 = 1$

d) $\frac{4^{-10}(4^{-3})^6}{(4^{-4})^8}$ $= (4)^{-10+(-3)(6)-(-4)(8)} = 4^{-10-18+32} = 4^4 = 256$

e) $(11)^9\left(\frac{1}{11}\right)^7$ $= 11^9(11)^{-7} = 11^{9+(-7)} = 11^2 = 121$

f) $\left(\frac{(-3)^7(-3)^4}{(-3^4)^3}\right)^{-3} = (-3)^{7+4-(4)(3)})^{-3} = ((-3)^{-1})^{-3} = 3^3 = 27$

May 15-11:06

3. Express each radical in exponential form and each power in radical form.

a) $\sqrt[3]{x^7}$ $= x^{7/3}$

c) $(\sqrt{p})^{11}$ $= p^{11/2}$

b) $y^{\frac{8}{5}}$ $= (\sqrt[5]{y})^8$

d) $m^{1.25}$ $= m^{5/4}$ $= (\sqrt[4]{m})^5$

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4. Evaluate. Express answers in rational form.

a) $\left(\frac{2}{5}\right)^{-3} = \left(\frac{5}{2}\right)^{3} = \frac{125}{8}$

b) $\left(\frac{16}{225}\right)^{-0.5} = \left(\frac{225}{16}\right)^{0.5} = \frac{\sqrt{225}}{\sqrt{16}} = \frac{15}{4}$

c) $\frac{(81)^{-0.25}}{\sqrt[3]{-125}} = \left(\frac{1}{81}\right)^{0.25} \div (-5) = \frac{1}{3} \div -5 = -\frac{1}{15}$

d) $(\sqrt[3]{-27})^4 = (-3)^4 = 81$

e) $(\sqrt[5]{-32})(\sqrt[6]{64})^5 = (-2)(2)^5 = (-2)(32) = -64$

f) $\sqrt[6]{((-2)^3)^2} = \left((-8)^2\right)^{1/6} = (-8)^{1/3} = \sqrt[3]{-8} = -2$

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5. Simplify. Write with only positive exponents.

a) $a^{\frac{3}{2}}\left(a^{-\frac{3}{2}}\right) = a^{3/2+(-3/2)} = a^0 = 1$

b) $\frac{b^{0.8}}{b^{-0.2}} = b^{0.8-(-0.2)} = b^1 = b$

c) $\frac{c\left(c^{\frac{5}{6}}\right)}{c^2} = c^{1+5/6-2} = c^{-1/6} = \frac{1}{c^{1/6}}$

d) $\frac{d^{-5}d^{\frac{11}{2}}}{(d^{-3})^2} = d^{-5+11/2-(-3)(2)} = d^{1/2+6} = d^{13/2}$

e) $\left((e^{-2})^{\frac{7}{2}}\right)^{-2} = e^{-2(7/2)(-2)} = e^{14}$

f) $\left((f^{-\frac{1}{6}})^{\frac{6}{5}}\right)^{-1} = f^{-\frac{1}{6}\left(\frac{6}{5}\right)(-1)} = f^{1/5}$

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7. Evaluate each expression for the given values. Express answers in rational form.

a) $(5x)^2(2x)^3; x = -2$

b) $\frac{8m^{-5}}{(2m)^{-3}}; m = 4$

c) $\frac{2w(3w^{-2})}{(2w)^2}; w = -3$

d) $\frac{(9y)^2}{(3y^{-1})^3}; y = -2$

e) $(6(x^{-4})^3)^{-1}; x = -2$

f) $\frac{(-2x^{-2})^3(6x)^2}{2(-3x^{-1})^3}; x = \frac{1}{2}$

a) $= (25x^2)(8x^3)$
$= 400x^5$
$\Rightarrow 400(-2)^5$
$= -6400$

b) $= \frac{8m^{-5}}{\frac{1}{8m^3}}$
$= 8m^{-5} \times 8m^3$
$= 64m^{-2}$
$\Rightarrow 64(4)^{-2}$
$= 4$

c) $= \frac{6w^{-1}}{4w^2}$
$= \frac{3}{2w^3}$
$= \frac{3}{2(-3)^3}$
$= \frac{3}{-54} = -\frac{1}{18}$

d) $\frac{81y^2}{27y^{-3}} = 3y^5$
$= 3(-2)^5$
$= 3(-32)$
$= -96$

e) $= (6x^{-4(3)})^{-1}$
$= (6x^{-12})^{-1}$
$= \frac{x^{12}}{6}$
$\Rightarrow (-2)^{12} \div 6$
$= \frac{4096}{6} = \frac{2048}{3}$

f) $= \frac{(-8x^{-6})(36x^2)}{2(-27x^{-3})}$
$= \frac{16}{3}x^{-6+2-(-3)}$
$= \frac{16}{3x}$
$\Rightarrow \frac{16}{3(\frac{1}{2})} = \frac{16}{3\frac{1}{2}}$
$= 16 \times \frac{2}{3} = \frac{32}{3}$

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9. Identify the type of function (linear, quadratic, or exponential) for each table of values.

a)

x	y
−5	−38
0	−3
5	42
10	97
15	162
20	237

$\rightarrow 35 \rightarrow 10$
$\rightarrow 45 \rightarrow 10$
$\rightarrow 55 \rightarrow 10$
$\rightarrow 65 \rightarrow 10$
$\rightarrow 75 \rightarrow 10$

2ND differences are constant $\Rightarrow$ Quadratic

c)

x	y
1	13
2	43
3	163
4	643
5	2 563
6	10 243

$\rightarrow 30 \rightarrow \times 4$
$\rightarrow 120 \rightarrow \times 4$
$\rightarrow 480 \rightarrow \times 4$
$\rightarrow 1920 \rightarrow \times 4$
$\rightarrow 7680$

Common ratio is constant $\Rightarrow$ Exponential

d)

x	y
−2	40
−1	20
0	10
1	5
2	2.5
3	1.25

$\rightarrow \times \frac{1}{2}$
$\rightarrow \times \frac{1}{2}$
$\rightarrow \times \frac{1}{2}$
$\rightarrow \times \frac{1}{2}$
$\rightarrow \times \frac{1}{2}$

Common ratio is constant $\Rightarrow$ Exponential

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9. Identify the type of function (linear, quadratic, or exponential) for each table of values.

b)

x	y	
0	−45	
2	−15	> 30
4	15	> 30
6	45	> 30
8	75	> 30
10	105	> 30

1ST differences are constant
⇒ Linear

e)

x	y	
−2	2000	
−1	1000	> x ½
0	500	> x ½
1	250	> x ½
2	125	> x ½
3	62.5	> x ½

Common ratio is constant
⇒ Exponential

f)

x	y	
0.2	−10.8	
0.4	−9.6	> 1.2 > x2
0.6	−7.2	> 2.4 > x2
0.8	−2.4	> 4.8 > x2
1	7.2	> 9.6 > x2
1.2	26.4	> 19.2

Common ratio is constant
⇒ Exponent

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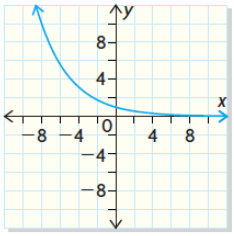
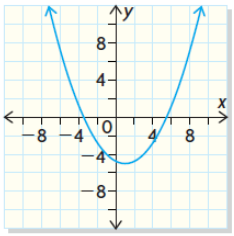
10. Identify each type of function (linear, quadratic, or exponential) from its graph.

a)



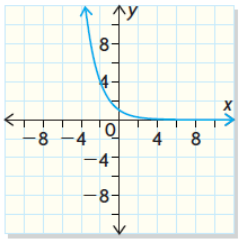
Exponential

b)



Quadratic

c)



Exponential

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11. For each exponential function, state the base function, $y = b^x$. Then state the transformations that map the base function onto the given function. Use transformations to sketch each graph.

a) $y = \left(\frac{1}{2}\right)^{\frac{x}{2}} - 3$

Base is $y = \left(\frac{1}{2}\right)^x$

HS factor of 2 $\left(\frac{1}{k}\right)$

VT down 3 (c)

$(0,1) \rightarrow (0,-2)$

$(1,\frac{1}{2}) \rightarrow (2,-2\frac{1}{2})$

$x \rightarrow \frac{x}{k} + d$

$y \rightarrow ay + c$

b) $y = \frac{1}{4}(2)^{-x} + 1$

Base is $y = 2^x$

VC factor of $\frac{1}{4}$ (a)

Reflect in y-axis

VT up 1 (c)

$(0,1) \rightarrow (0,1\frac{1}{4})$

$(1,2) \rightarrow (-1,1\frac{1}{2})$

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11. For each exponential function, state the base function, $y = b^x$. Then state the transformations that map the base function onto the given function. Use transformations to sketch each graph.

c) $y = -2(3)^{2x+4}$

Base is $y = 3^x$

$y = -2(3)^{2(x+2)}$

VS factor of 2 (a)

Reflect in x-axis

HC factor of $\frac{1}{2}$ $\left(\frac{1}{k}\right)$

HT left 2 (d)

$(0,1) \rightarrow (-2,-2)$

$(1,3) \rightarrow (-1\frac{1}{2},-6)$

d) $y = \frac{-1}{10}(5)^{3x-9} + 10$

Base is $y = 5^x$

$y = -\frac{1}{10}(5)^{3(x-3)} + 10$

VC factor of $\frac{1}{10}$ (a)

Reflect in x-axis

HC factor of $\frac{1}{3}$ $\left(\frac{1}{k}\right)$

HT right 3 (d)

VT up 10 (c)

$(0,1) \rightarrow (3,9\frac{9}{10})$

$(1,5) \rightarrow (3\frac{1}{3},9\frac{1}{2})$

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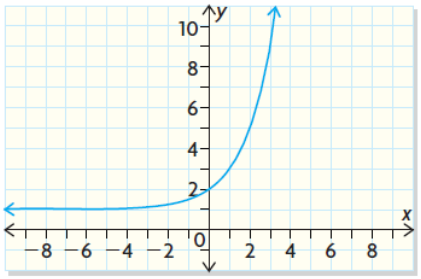
12. The exponential function shown has been reflected in the y-axis and translated vertically. State its y-intercept, its asymptote, and a possible equation for it.



IGNORE

Not on the test

You're welcome

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13. Complete the table.

	Function	Exponential Growth or Decay?	Initial Value (y-intercept)	Growth or Decay Rate
a)	$V(t) = 100(1.08)^t$	Growth	100	8%
b)	$P(n) = 32(0.95)^n$	Decay	32	5%
c)	$A(x) = 5(3)^x$	Growth	5	200%
d)	$Q(n) = 600\left(\frac{5}{8}\right)^n$	Decay	600	37.5%

Growth rate $= (\text{Base} - 1) \times 100\%$

Decay rate $= (1 - \text{Base}) \times 100\%$

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14. A hot cup of coffee cools according to the equation

$$T(t) = 69\left(\frac{1}{2}\right)^{\frac{t}{30}} + 21$$

where T is the temperature in degrees Celsius and t is the time in minutes.

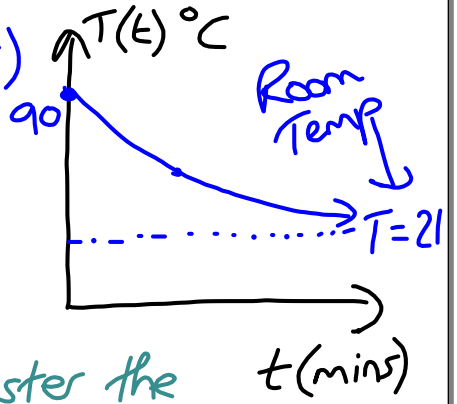
a) Which part of the equation indicates that this is an example of exponential decay?
b) What was the initial temperature of the coffee?
c) Use your knowledge of transformations to sketch the graph of this function.
d) Determine the temperature of the coffee, to the nearest degree, after 48 min.
e) Explain how the equation would change if the coffee cooled faster.
f) Explain how the graph would change if the coffee cooled faster.

a) Base is $0<b<1$

b) $T(0) = 69+21 = 90°C$

d) $T(48) = 69\left(\frac{1}{2}\right)^{48/30} + 21$
$= 43.76$
$= 44°C$

c)



e) If the coffee cooled faster the k value ($\frac{1}{30}$) would be smaller

f) The graph would be horizontally compressed

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15. The value of a car after it is purchased depreciates according to the formula

$$V(n) = 28\,000(0.875)^n$$

where $V(n)$ is the car's value in the nth year since it was purchased.

a) What is the purchase price of the car?
b) What is the annual rate of depreciation?
c) What is the car's value at the end of 3 years?
d) What is its value at the end of 30 months?
e) How much value does the car lose in its first year?
f) How much value does it lose in its fifth year?

a) $28,000

b) Rate $= (1-0.875)\times 100\%$
$= 12.5\%$

c) $V(3) = 28000(0.875)^3$
$= \$18,757.81$

d) 30 months $= \frac{30}{12}$ years
$= 2.5$
$V(2.5) = 28000(0.875)^{2.5}$
$= \$20,052.95$

e) $= V(0) - V(1)$
$= 28000 - 24500$
$= \$3500$

f) $= V(4) - V(5)$
$= 16443.09 - 14361.45$
$= \$2051.64$

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16. Write the equation that models each situation. In each case, describe each part of your equation.
   a) the percent of a pond covered by water lilies if they cover one-third of a pond now and each week they increase their coverage by 10%
   b) the amount remaining of the radioactive isotope U_{238} if it has a half-life of 4.5×10^9 years
   c) the intensity of light if each gel used to change the colour of a spotlight reduces the intensity of the light by 4%

b) $A(t) = A_0\left(\frac{1}{2}\right)^{t/4.5\times10^9}$ (time in years; $A(t)$: amount after t years; A_0: initial amount; 4.5×10^9: half-life)

a) Initial coverage $= \frac{1}{3}$
$= 33\frac{1}{3}\%$

Growth rate $= 10\%$
$\Rightarrow$ base $= 1+\frac{10}{100}$
$= 1.1$

$\Rightarrow P(w) = 33\frac{1}{3}(1.1)^w$
where w = # of weeks
and $P(w)$ = % covered

c) $I(g) = 100(0.96)^g$
g = # of gels
$I(g)$ = intensity
100 = initial intensity
0.96 = decay of 4%

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17. The population of a city is growing at an average rate of 3% per year. In 1990, the population was 45 000.
   a) Write an equation that models the growth of the city. Explain what each part of the equation represents.
   b) Use your equation to determine the population of the city in 2007.
   c) Determine the year during which the population will have doubled.
   d) Suppose the population took only 10 years to double. What growth rate would be required for this to have happened?

a) $P(t) = 45000(1.03)^t$ (# of years since 1990; $P(t)$: population after t years since 1990; 45000: initial population in 1990; 1.03: growth rate of 3%)

b) $t = 2007 - 1990$
$t = 17$
$P(17) = 45000(1.03)^{17}$
$= 74378$ people

c) $90000 = 45000(1.03)^t$
$2 = 1.03^t$
Guess and check
$\Rightarrow t = 23.4$
Year $= 1990 + 23.4$
$= 2013.4$

d) $2 = x^{10}$
where x = base
$\sqrt[10]{2} = x$
$2^{0.1} = x$
$1.07 = x$
$\Rightarrow$ rate $= (1.07-1)\times100\%$ $= 7\%$ growth rate

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