

Practise, Apply, Solve 1.11

A

1. Express each as a power of 3.

(a) 27 (b) 81 (c) $\frac{1}{9}$ (d) 9^{2x} (e) $\left(\frac{1}{27}\right)^x$

2. Determine which value of x is the solution to the equation.

(a) $3^{2x-5} = 27$ (b) $4^{x+3} = 64$ (c) $5^{x+2} = \frac{1}{25}$ (d) $2^{5x+2} = \sqrt{2}$
 i. $x = 1$ i. $x = 1$ i. $x = 0$ i. $x = -\frac{3}{10}$
 ii. $x = 4$ ii. $x = 0$ ii. $x = -4$ ii. $x = -\frac{1}{2}$

3. Determine the exact solutions algebraically.

(a) $2^x = 2^7$ (b) $5^x = 5^3$ (c) $3^{x+6} = 3^{12}$ (d) $10^{2x-1} = 10^3$
 (e) $2^{2x-1} = 2^{x+9}$ (f) $7^{3x+2} = 7^{2x+5}$ (g) $4^{2x} = 4^8$ (h) $5^x = 5^{3x-12}$

4. Find the exact roots of each equation.

(a) $2^x = 32$ (b) $3^x = 27$ (c) $3^x = 9^{x-1}$ (d) $5^x = 3125$
 (e) $4(2^x) = 32$ (f) $5^x = \frac{1}{125}$ (g) $6^x = \sqrt[3]{6}$ (h) $3^{-x} = \frac{1}{81}$

5. Determine the approximate solutions using your calculator and guessing and checking. Round to two decimal places.

(a) $2^x = 20$ (b) $5^x = 35$ (c) $3^x = 100$ (d) $10^x = 800$
 (e) $2^{2x} = 50$ (f) $7^{3x} = 150$ (g) $4^{2x-1} = 80$ (h) $200 = 5^{3x+2}$

B

For questions 6 to 9, determine the exact solutions algebraically.

6. Solve each equation without using a calculator.

(a) $4^x - 8\sqrt{2}$ (b) $3^x - \sqrt[5]{9}$ (c) $125^x = 25\sqrt{5}$ (d) $8x = 16\sqrt[3]{2}$

7. (a) Solve. $\left(\frac{1}{9}\right)^{x+2} = \left(\frac{1}{27}\right)^{x+3}$

(b) Verify your answer in (a).

8. Solve each equation.

(a) $2^{7-x} = \frac{1}{2}$ (b) $2^{x-2} = 4^{x+2}$ (c) $\left(\frac{1}{4}\right)^{x-2} = \left(\frac{1}{8}\right)^{x+1}$
 (d) $9^{2x+1} = 81(27^x)$ (e) $2^{2x+2} + 7 = 71$ (f) $(2^{x+1})(4^{x+1})(8^{x+1}) = 128^x$

9. Determine the solution or solutions of each equation.

(a) $2^{x^2} = 32(2^{4x})$ (b) $3^{x^2} = 27(3^{2x})$ (c) $9^{x+2} = \left(\frac{1}{27}\right)^{x+2}$
 (d) $2^{r^2+6r} = 2^{-8}$ (e) $3^{x^2+20} = \left(\frac{1}{27}\right)^{3x}$ (f) $(8)^{x^2} = (4)^{4-5x}$
 (g) $(2^{x-4})^x = 32$ (h) $9^{x^2+1} = (27^x)(3^{2x})$ (i) $2^{x^2} = (16^{x-1})(2^x)$

10. Determine the number of terms in each geometric sequence.
- (a) 2, 6, 18, ... , 39 366 (b) 5, 10, 20, ... , 10 240
(c) -2, -10, -50, ... , -6250 (d) 16, 8, 4, ... , $\frac{1}{64}$
(e) 3, 6, 12, ... , 384 (f) 4, 24, 144, ... , 186 624
11. Determine the approximate solutions using graphing technology. Round to two decimal places.
- (a) $3^x = 30$ (b) $5^z = 10$ (c) $5.6^y = 60$ (d) $(1.04)^x = 2$
(e) $25^{-x} = 10$ (f) $12^{2x} = 500$ (g) $4.1^{3x} = 40$ (h) $5^{2x-1} = 45$
12. A bacteria culture doubles in size every 15 min. How long will it take for a culture of 20 bacteria to grow to a population of 163 840?
13. **Knowledge and Understanding:** Determine the exact solution of $9^{2x+1} = 81(27^x)$.
14. The use of wind turbines to generate electrical energy in Europe has increased exponentially. The energy produced by wind turbines between 1980 and 1995 can be modelled by the equation $y = 6.489(1.580)^x$, where x is the number of years since 1980 and y is the number of gigawatt-hours of energy produced.
- (a) Determine the amount of energy produced in 1980.
(b) Determine the amount of energy produced in 1992.
(c) In what year was 398.191 107 2 GW-h produced?
(d) Determine when 500 GW-h of energy were produced.
15. If \$500 is deposited in an account paying 8%/a, compounded semiannually, how long will it take for the deposit to increase to \$900?
16. **Communication:** You can solve an exponential equation algebraically or by using a graph. Explain how you would decide which method to use. Include examples.
17. Thorium-227 has a half-life of 18.4 days. How much time will a 50-mg sample take to decompose to 10 mg?
18. **Thinking, Inquiry, Problem Solving:** When a plant or an animal dies, it stops absorbing carbon-14 from the atmosphere. Carbon-14 is an unstable radioactive isotope and decays over time. By measuring the amount of carbon-14 remaining in a sample from a plant or animal fossil, scientists can accurately predict the age of the specimen.
- (a) Research to find the half-life of carbon-14.
(b) Estimate the age of the fossil of a leaf that contains 0.10% of the original amount of carbon-14.

