Solutions

1. Express each as a power of 3.

(a) 27 (b) 81 (c) $\frac{1}{9}$ (d) 9^{2x} (e) $\left(\frac{1}{27}\right)^x$

(a) $= 3^3$

(b) $= 3^4$

(c) $= 3^{-2}$

(d) $= (3^2)^{2x}$
$= 3^{4x}$

(e) $= (3^{-3})^x$
$= 3^{-3x}$

2. Determine which value of x is the solution to the equation.

(a) $3^{2x-5} = 27$

i. $x = 1$

ii. $x = 4$

$3^{2x-5} = 3^3$

$\Rightarrow 2x - 5 = 3$

$\frac{2x}{2} = \frac{8}{2}$

$x = 4$

(c) $5^{x+2} = \frac{1}{25}$

i. $x = 0$

ii. $x = -4$

$5^{x+2} = 5^{-2}$

$\Rightarrow x + 2 = -2$

$x = -4$

3. Determine the exact solutions algebraically.

(a) $2^x = 2^7$

$\Rightarrow x = 7$

(c) $3^{x+6} = 3^{12}$

$\Rightarrow x + 6 = 12$

$x = 6$

(e) $2^{2x-1} = 2^{x+9}$

$\Rightarrow 2x - 1 = x + 9$

$x - 1 = 9$

$x = 10$

(g) $4^{2x} = 4^8$

$\Rightarrow \frac{2x}{2} = \frac{8}{2}$

$x = 4$

4. Find the exact roots of each equation.

(a) $2^x = 32$

$$2^x = 2^5$$
$$\Rightarrow x = 5$$

(c) $3^x = 9^{x-1}$

$$3^x = (3^2)^{x-1}$$
$$3^x = 3^{2x-2}$$
$$\Rightarrow x = 2x - 2$$
$$2 = 2x - x$$
$$2 = x$$

(e) $4(2^x) = 32$

$$\frac{4(2^x)}{4} = \frac{32}{4}$$
$$2^x = 8$$
$$2^x = 2^3$$
$$\Rightarrow x = 3$$

(g) $6^x = \sqrt[3]{6}$

$$6^x = 6^{1/3}$$
$$\Rightarrow x = \frac{1}{3}$$

6. Solve each equation without using a calculator.

(a) $4^x = 8\sqrt{2}$

$$(2^2)^x = (2^3)(2^{1/2})$$
$$2^{2x} = 2^{7/2}$$
$$\Rightarrow \frac{2x}{2} = \frac{\frac{7}{2}}{2}$$
$$x = \frac{7}{4}$$

(c) $125^x = 25\sqrt{5}$

$$(5^3)^x = (5^2)(5^{1/2})$$
$$5^{3x} = 5^{5/2}$$
$$\Rightarrow \frac{3x}{3} = \frac{\frac{5}{2}}{3}$$
$$x = \frac{5}{6}$$

8. Solve each equation.

(a) $2^{7-x} = \frac{1}{2}$ (e) $2^{2x+2} + 7 = 71$ (c) $\left(\frac{1}{4}\right)^{x-2} = \left(\frac{1}{8}\right)^{x+1}$

$2^{7-x} = 2^{-1}$

$\Rightarrow 7 - x = -1$

$8 = x$

$2^{2x+2} = 64$

$2^{2x+2} = 2^6$

$\Rightarrow 2x+2 = 6$

$\frac{2x}{2} = \frac{4}{2}$

$x = 2$

$(2^{-2})^{x-2} = (2^{-3})^{x+1}$

$2^{-2x+4} = 2^{-3x-3}$

$\Rightarrow -2x+4 = -3x-3$

$3x - 2x = -3 - 4$

$x = -7$

9. Determine the solution or solutions of each equation.

(a) $2^{x^2} = 32(2^{4x})$ (e) $3^{x^2+20} = \left(\frac{1}{27}\right)^{3x}$ (c) $9^{x+2} = \left(\frac{1}{27}\right)^{x+2}$

$2^{x^2} = 2^5(2^{4x})$

$2^{x^2} = 2^{5+4x}$

$\Rightarrow x^2 = 5 + 4x$

$x^2 - 4x - 5 = 0$

$(x-5)(x+1) = 0$

$x = 5$ or -1

$3^{x^2+20} = (3^{-3})^{3x}$

$3^{x^2+20} = 3^{-9x}$

$\Rightarrow x^2 + 20 = -9x$

$x^2 + 9x + 20 = 0$

$(x+4)(x+5) = 0$

$x = -4$ or -5

$(3^2)^{x+2} = (3^{-3})^{x+2}$

$3^{2x+4} = 3^{-3x-6}$

$\Rightarrow 2x+4 = -3x-6$

$2x + 3x = -6 - 4$

$\frac{5x}{5} = \frac{-10}{5}$

$x = -2$

15. If \$500 is deposited in an account paying 8%/a, compounded ~~semi~~annually, how long will it take for the deposit to increase to \$900?

$$A = A_0(b)^n$$

$A = \$900$

$A_0 = \$500$

$b = 1 + \frac{8}{100}$

$= 1.08$

$n = ?$

$$\frac{900}{500} = \frac{500(1.08)^n}{500}$$

$$1.8 = (1.08)^n$$

Guess and check!

$n = 7 \Rightarrow 1.08^7 = 1.714$

$n = 8 \Rightarrow 1.08^8 = 1.851$

$n = 7.5 \Rightarrow 1.08^{7.5} = 1.781$

$n = 7.6 \Rightarrow 1.08^{7.6} = 1.795$

$n = 7.7 \Rightarrow 1.08^{7.7} = 1.809$

$n = 7.6$ gives closest answer to $1.8 \Rightarrow 7.6$ years

17. Thorium-227 has a half-life of 18.4 days. How much time will a 50-mg sample take to decompose to 10 mg?

$$A = A_0(b)^{t/h}$$

$A = 10$ mg

$A_0 = 50$ mg

$b = \frac{1}{2}$

$t/h = t/18.4$

Let $x = \frac{t}{18.4}$

$$\frac{10}{50} = \frac{50\left(\frac{1}{2}\right)^{t/18.4}}{50}$$

$$0.2 = \left(\frac{1}{2}\right)^x$$

Guess and check

$x = 2 \Rightarrow \left(\frac{1}{2}\right)^2 = 0.25$

$x = 3 \Rightarrow \left(\frac{1}{2}\right)^3 = 0.125$

$x = 2.3 \Rightarrow \left(\frac{1}{2}\right)^{2.3} = 0.203$

$x = 2.2 \Rightarrow \left(\frac{1}{2}\right)^{2.2} = 0.218$

$x = 2.4 \Rightarrow \left(\frac{1}{2}\right)^{2.4} = 0.189$

$x = 2.3$ gives closest answer to 0.2

However $x = \frac{t}{18.4} \Rightarrow t = 2.3(18.4)$

$t = 42.3$ days

2. Complete the table.

	Function	Exponential Growth or Decay?	Initial Value	Growth or Decay Rate
a)	$V(t) = 20(1.02)^t$	Growth	20	2%
b)	$P(n) = (0.8)^n$	Decay	1	20%
c)	$A(x) = 0.5(3)^x$	Growth	0.5	200%
d)	$Q(w) = 600\left(\frac{5}{8}\right)^w$	Decay	600	37.5%

Growth rate $= (\text{base} - 1) \times 100\%$

Decay rate $= (1 - \text{base}) \times 100\%$

3. The growth in population of a small town since 1996 is given by the function $P(n) = 1250(1.03)^n$.

a) What is the initial population? Explain how you know.

b) What is the growth rate? Explain how you know.

c) Determine the population in the year 2007.

d) In which year does the population reach 2000 people?

a) 1250. Sub in $n = 0$.

b) 3%. $(\text{Base} - 1) \times 100\%$

c) $2007 - 1996 = 11$

$P(11) = 1250(1.03)^{11}$

$= 1730$

d) $\dfrac{2000}{1250} = \dfrac{1250(1.03)^n}{1250}$

$1.6 = (1.03)^n$

using guess and check $\Rightarrow$ 15.9 years

$\Rightarrow 1996 + 15.9 = 2011.9$

So during 2012

4. A computer loses its value each month after it is purchased. Its value as a function of time, in months, is modelled by $V(m) = 1500(0.95)^m$.

a) What is the initial value of the computer? Explain how you know.

b) What is the rate of depreciation? Explain how you know.

c) Determine the value of the computer after 2 years.

d) In which month after it is purchased does the computer's worth fall below \$900?

a) \$1500. Find $V(0)$.

b) 5%. $(1-\text{Base}) \times 100\%$.

c) 2 years $= 2(12)$

$= 24$ months

$V(24) = 1500(0.95)^{24}$

$= \$437.98$

d) $\frac{900}{1500} = \frac{1500(0.95)^n}{1500}$

$0.6 = (0.95)^n$

using guess and check $\Rightarrow n = 9.96$

$\Rightarrow$ 10 months

5. In 1990, a sum of \$1000 is invested at a rate of 6% per year for 15 years.

a) What is the growth rate?

b) What is the initial amount?

c) How many growth periods are there?

d) Write an equation that models the growth of the investment, and use it to determine the value of the investment after 15 years.

a) 6%

b) \$1000

c) 15

d) $A(n) = 1000(1.06)^n$

$A(15) = 1000(1.06)^{15}$

$= \$2396.56$

10. In each case, write an equation that models the situation described. Explain what each part of each equation represents.

a) the percent of colour left if blue jeans lose 1% of their colour every time they are washed

$$A(n) = 100(0.99)^n$$

n — # of washes

100 — starting % of colour

0.99 — decay rate of 1%

b) the population if a town had 2500 residents in 1990 and grew at a rate of 0.5% each year after that for *t* years

$$P(t) = 2500(1.005)^t$$

t — # of years since 1990

P(t) — population

2500 — initial population

1.005 — growth rate of 0.5%

c) the population of a colony if a single bacterium takes 1 day to divide into two; the population is *P* after *t* days

$$P(t) = 1(2)^t$$

t — # of days

P(t) — population

1 — initial population

2 — growth rate of 100% (doubling)