Warm up

Explain the transformations to the function $y = b^x$

a) $f(x) = 3b^{x+1}$

VS factor of 3

HT left 1

b) $g(x) = 0.5b^{2x} - 1$

VC factor of 0.5

HC factor of $\frac{1}{2}$ $\left(\frac{1}{k}\right)$

VT down 1

Solving Exponential Equations

Lesson objectives

- I can find like bases to solve an exponential equation
- I can solve an exponential equation that acts as a quadratic
- I know the general form of an exponential and can write it using an equation
- I understand what the base of a growth problem looks like
- I understand what the base of a decay problem looks like

1.1

Lesson objectives | Teachers' notes | Lesson notes

Handout #s 1, 2ac, 3aceg, 4aceg, 6ac, 8ace, 9ace, 15 & 17

AND Nelson Page 261 #s 2 - 5 & 10

Example

A 200g sample of radioactive polonium-210 has a half-life of 138 days. This means that every 138 days, the amount of polonium left in a sample is half of the original amount. The mass of polonium, in grams, that remains after t days can be modelled by

$$m(t) = 200(0.5)^{t/138}$$

a) Determine the mass that remains after 5 years.

$t = 5(365)$

$t = 1825$

$\Rightarrow m(1825) = 200(0.5)^{\frac{1825}{138}}$

$= 0.02g$

b) How long does it take for the sample of 200g to decay to 12.5 grams?

200g → 100g
100g → 50g
50g → 25g
25g → 12.5g

4 half-life cycles

$\Rightarrow t = 4(138)$

$t = 552$ days

Equivalent Exponential Expressions

We saw before with polynomial expressions that we can prove expressions are **NOT** equal by giving a counter-example, but we need to show the equations are the same to prove that they are equivalent.

We have the same ideas for exponential functions.

Eg Show $2^{2x} = 4^x$

x	2^{2x}	4^x
0	$2^{2(0)} = 1$	$4^0 = 1$
1	$2^{2(1)} = 4$	$4^1 = 4$
2	$2^{2(2)} = 16$	$4^2 = 16$
3	$2^{2(3)} = 64$	$4^3 = 64$

Looking at the table of values on the left we can see that many values are the same, but we need to prove the equations are.

Equivalent Bases

First we need to show that they have the same base.

2^{2x} 4^{x}

We know that $4 = 2^2$ therefore we could have a base of 2

We then substitute so they are expressed as the same base

$2^{2x} = (2^2)^x$

Using the power of a power law we

$2^{2x} = 2^{2x}$ ← power of a power law

and we can see that we have equivalent expressions because the equations are the same

Example

Show that the following are equivalent:

a) $3^{6x} = 9^{3x}$

Since $9 = 3^2$

$\Rightarrow 3^{6x} = (3^2)^{3x}$

$3^{6x} = 3^{6x}$

b) $8^{4x} = 16^{3x}$

$8 = 2^3$, $16 = 2^4$

$\Rightarrow (2^3)^{4x} = (2^4)^{3x}$

$2^{12x} = 2^{12x}$

c) $(\sqrt{3})^2 = (1/3)^{-1}$

$\sqrt{3} = 3^{1/2}$, $\frac{1}{3} = 3^{-1}$

$\Rightarrow (3^{1/2})^2 = (3^{-1})^{-1}$

$3^1 = 3^1$

d) $3^{2x} = 81^{0.5x}$

$81 = 3^4$

$\Rightarrow 3^{2x} = (3^4)^{0.5x}$

$3^{2x} = 3^{2x}$

Solving Exponential Functions

We can also use this method of equivalent expressions to solve problems.

If we know the expressions are equivalent and we have the same base, then the exponents must be the same

Eg Solve for x, given that $3^{2x} = 81$

$3^4 = 81$

$\Rightarrow 3^{2x} = 3^4$

If bases are equal, then exponents are equal

So, $\Rightarrow \frac{2x}{2} = \frac{4}{2}$

$x = 2$

Example

Solve for x

a) $4^x = 16$

$16 = 4^2$

$\Rightarrow 4^x = 4^2$

So, $x = 2$

b) $9^{2x} = 27^3$

$9 = 3^2$, $27 = 3^3$

$\Rightarrow (3^2)^{2x} = (3^3)^3$

$3^{4x} = 3^9$

So, $\frac{4x}{4} = \frac{9}{4}$

$x = 2.25$

c) $4^{2x+1} = 2^4$

$4 = 2^2$

$\Rightarrow (2^2)^{2x+1} = 2^4$

$2^{4x+2} = 2^4$

So, $4x + 2 = 4$

$\frac{4x}{4} = \frac{2}{4}$

$x = 0.5$

Solving using Factoring

We also could come across scenarios that require us to use quadratic factoring.

Example

Find all possible values of x for $2^{x^2+3x} = \frac{1}{4}$

$\frac{1}{4} = 4^{-1} = (2^2)^{-1} = 2^{-2}$

$\Rightarrow 2^{x^2+3x} = 2^{-2}$

So, $x^2 + 3x = -2$

$x^2 + 3x + 2 = 0$

$(x+1)(x+2) = 0$

$\Rightarrow x = -1, x = -2$

Half-Life Problems

Radioactive material decays (half-life problems) according to the relationship

$$A(t) = A_o(0.5)^{t/h}$$

where A(t) is the mass of the decayed material
A_o is the mass of the original material
t is the time
h is the half-life

When solving these problems for time we need to use the strategies involved with equivalent expressions.

Exponential Growth

This can be modelled by

$$A(t) = A_o(b)^t$$

where A(t) is the amount after growth
A_o is the original amount
b is the growth rate per time period
t is the number of growth periods

NOTE: For growth problems, b **MUST** be greater than 1.

i.e. If growing by 5%, then b = 1 + 0.05 = 1.05

Exponential Decay

This can be modelled by

$$A(t) = A_o(b)^t$$

where A(t) is the amount after growth
A_o is the original amount
b is the growth rate per time period
t is the number of growth periods

NOTE: For decay problems, b **MUST** be less than 1.

i.e. If decaying by 5%, then b = 1 - 0.05 = 0.95
If it decays by 5% then 95% remains.

Example

Bacteria quadruple in population every 6 hours

a) If you had 100 bacteria in a petrie dish for 100 hours how many bacteria would be present?

$A_0 = 100$

$b = 4$

$t = 100/6$

[# of growth cycles]

$A\left(\frac{100}{6}\right) = 100(4)^{100/6}$

$A = 1.08 \times 10^{12}$

b) If you started with 100 bacteria and after a certain time had 25600, how long would have the bacteria been present?

$$\frac{25600}{100} = \frac{100(4)^{t/6}}{100}$$

$$256 = 4^{t/6}$$

$[256 = 4^4]$

$$4^4 = 4^{t/6}$$

So, $4 = t/6 \Rightarrow 6(4) = t = 24$ hours

Example

Bunter bought a car for \$18000. The value of the car depreciates by 20% every year.

a) Write the equation that represents this relationship.

$A_0 = 18000$

$b = \left(1 - \frac{20}{100}\right) = 0.80$

$t = $ # of years

$A(t) = 18000(0.80)^t$

b) If Bunter owns the car for 4 years, what is the car's value.

Find $A(4)$

$$\Rightarrow A(4) = 18000(0.80)^4$$

$$= \$7372.80$$