6.60 Exponential Growth and Decay and Solving

Warm up

Explain the transformations to the function $y = b^x$

a) $f(x) = 3b^{x+1}$

b) $g(x) = 0.5b^{2x} - 1$

Example

A 200g sample of radioactive polonium-210 has a half-life of 138 days. This means that every 138 days, the amount of polonium left in a sample is half of the original amount. The mass of polonium, in grams, that remains after t days can be modelled by

$$m(t) = 200\left(\frac{1}{2}\right)^{\frac{t}{138}}$$

a) Determine the mass that remains after 5 years

b) How long does it take for the sample of 200g to decay to 12.5 grams?

We saw before with polynomial expressions that we can prove expressions are **NOT** equal by giving a counter-example, but we need to show the equations are the same to prove that they are equivalent.

We have the same ideas for exponential functions

i.e. Show $2^{2x} = 4^x$

x	$y = 2^{2x}$	$y = 4^x$
0		
1		
2		
3		

Equivalent Bases

First we need to show that they have the same base

2^{2x} 4^{x}

We know that 4 = _____ therefore we could have a base of ____

We then substitute so they are expressed as the same base.

Using the power of a power law we can simplify

and we can see that we have equivalent expressions because the equations are the same

Examples

Show that the following are equivalent:

a) $3^{6x} = 9^{3x}$

b) $8^{4x} = 16^{3x}$

c) $(\sqrt{3})^{2} = (1/3)^{-1}$

d) $3^{2x} = 81^{0.5x}$

Solving Equations

We can also use this method of equivalent expressions to solve problems

If we know the expressions are equivalent and we have the same base then the exponents must be the same

i.e. Solve for x, given $3^{2x} = 81$

Example

Solve for x

a) $4^{x} = 16$

b) $9^{2x} = 27^{3}$

c) $4^{2x+1} = 2^{4}$

We also could come across scenarios that require us to use quadratic factoring

Example
Find all possible values of x for $2^{x^2+3x} = \frac{1}{4}$.

Half-Life Problems
Radioactive material decays (half-life problems) according to the relationship

$$A(t) = A_0\left(\frac{1}{2}\right)^{\frac{t}{h}}$$

where $A(t)$ is the mass of the decayed material
A_0 is the mass of the original material
t is the time
h is the half-life

When solving these problems for time we need to use the strategies involved with equivalent expressions

Exponential growth problems can be modelled by

$$A(t) = A_0b^t$$

where $A(t)$ is the amount after growth
A_0 is the original amount
b is the growth rate per time period
t is the number of growth periods

NOTE: for growth problems b **MUST** be greater than 1. i.e. if growing by 5%, b = 1 + 0.05 = 1.05

Exponential decay problems can be modelled by

$$A(t) = A_0b^t$$

where A(t) is the amount after growth
A_0 is the original amount
b is the growth rate per time period
t is the number of growth periods

NOTE: for decay problems b **MUST** be less than 1, i.e. if decaying by 5%, b = 1 – 0.05 = 0.95
If it decays by 5% then 95% remains

Example

Bacteria quadruple in population every 6 hours

a) If you had 100 bacteria in a petrie dish for 100 hours how many bacteria would be present?

b) If you started with 100 bacteria and after a certain time had 25600, how long would have the bacteria been present?

Example

Bunter bought a car for $18000. The value of the car depreciates by 20% every year.

a) Write the equation that represents this relationship.

b) If Bunter owns the car for 4 years, what is the car's value.

Homework:

Handout #s 1, 2ac, 3aceg, 4aceg, 6ac, 8ace, 9ace, 15 & 17

AND Nelson Page 261 #s 2 – 5 & 10