Solutions

Nov 20-18:35

1. Each of the following are transformations of $f(x) = 3^x$. Describe each transformation.

a) $g(x) = 3^x + 3$ c) $g(x) = \frac{1}{3}(3^x)$

VT up 3 (c) VC by a factor of $\frac{1}{3}$ (a)

b) $g(x) = 3^{x+3}$ d) $g(x) = 3^{\frac{x}{3}}$

HT left 3 (d) HS by a factor of 3 $(\frac{1}{k})$

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2. For each transformation, state the base function and then describe the transformations in the order they could be applied.

a) $f(x) = -3(4^{x+1})$

Base $y = 4^x$

VS factor of 3

Reflect in x-axis

HT left 1

c) $h(x) = 7(0.5^{x-4}) - 1$

Base $y = 0.5^x$

VS factor of 7

HT right 4

VT down 1

b) $g(x) = 2\left(\frac{1}{2}\right)^{2x} + 3$

Base $y = \left(\frac{1}{2}\right)^x$

VS factor of 2

HC factor of $\frac{1}{2}$

VT up 3

d) $k(x) = 5^{3x-6}$

$k(x) = 5^{3(x-2)}$

Base $y = 5^x$

HC factor of $\frac{1}{3}$

HT right 2

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5. Let $f(x) = 4^x$. For each function that follows,

- state the transformations that must be applied to $f(x)$
- state the y-intercept and the equation of the asymptote
- sketch the new function
- state the domain and range

a) $g(x) = 0.5f(-x) + 2$

VC factor of 0.5

Reflect in y-axis

VT up 2

y-int at $g(0)$

$\Rightarrow 0.5(4)^{(-0)} + 2$

$= 0.5 + 2$

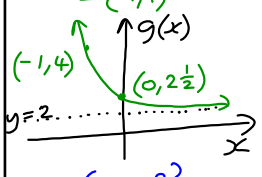
$= 2.5$

$(0,1) \rightarrow \begin{pmatrix} 0 \rightarrow \frac{0}{-1} = 0 \\ 1 \rightarrow 0.5(1)+2 = 2.5 \end{pmatrix}$

$= (0, 2.5)$

$(1,4) \rightarrow \begin{pmatrix} 1 \rightarrow \div -1 = -1 \\ 4 \rightarrow 0.5(4)+2 = 4 \end{pmatrix}$

$= (-1, 4)$



$D = \{x \in \mathbb{R}\}$

$R = \{y \in \mathbb{R} \mid y > 2\}$

c) $g(x) = -2f(2x - 6)$ $(2(x-3))$

VS factor of 2

Reflect in x-axis

HC factor of $\frac{1}{2}$ $\left(\frac{1}{k}\right)$

HT right 3

y-int at $g(0)$

$\Rightarrow -2(4)^{2(0-3)}$

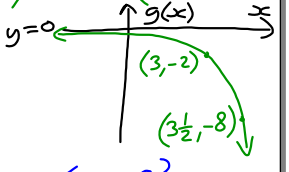
$= -2\left(\frac{1}{4096}\right)$

$= -\frac{1}{2048}$

$(0,1) \rightarrow \begin{pmatrix} 0 \rightarrow \frac{0}{2}+3 = 3 \\ 1 \rightarrow -2(1) = -2 \end{pmatrix}$

$= (3, -2)$

$(1,4) \rightarrow \begin{pmatrix} 1 \rightarrow \frac{1}{2}+3 = 3\frac{1}{2} \\ 4 \rightarrow -2(4) = -8 \end{pmatrix}$



$D = \{x \in \mathbb{R}\}$

$R = \{y \in \mathbb{R} \mid y < 0\}$

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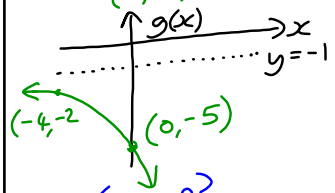
5. Let $f(x) = 4^x$. For each function that follows,
K
- state the transformations that must be applied to $f(x)$
- state the y-intercept and the equation of the asymptote
- sketch the new function
- state the domain and range

$(\frac{1}{4}(x+4))$ $(-\frac{1}{2}(x-2))$

b) $h(x) = -f(0.25x + 1) - 1$

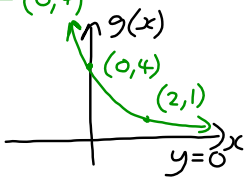
Reflect in x-axis
VS factor of 4 $(\frac{1}{k})$
HT left 4
VT down 1
y-int at $h(0)$
$\Rightarrow -4^{(\frac{1}{4}(0+4))}-1$
$= -5$
$(0,1) \to (0 \to 0/\frac{1}{4} - 4 = -4,\ 1 \to -(1) - 1 = -2)$
$= (-4,-2)$
$(1,4) \to (1 \to 1/\frac{1}{4} - 4 = 0,\ 4 \to -(4) - 1 = -5)$
$= (0,-5)$



$D = \{x \in \mathbb{R}\}$
$R = \{y \in \mathbb{R} \mid y < -1\}$

d) $h(x) = f(-0.5x + 1)$

Reflect in y-axis
VS factor of 2 $(\frac{1}{k})$
HT right 2
y-int at $h(0)$
$\Rightarrow 4^{(-\frac{1}{2}(0-2))}$
$= 4$
$(0,1) \to (0 \to 0/-\frac{1}{2} + 2 = 2,\ 1 \to 1 = 1)$
$= (2,1)$
$(1,4) \to (1 \to 1/-\frac{1}{2} + 2 = 0,\ 4 \to 4 = 4)$
$= (0,4)$



$D = \{x \in \mathbb{R}\}$
$R = \{y \in \mathbb{R} \mid y > 0\}$

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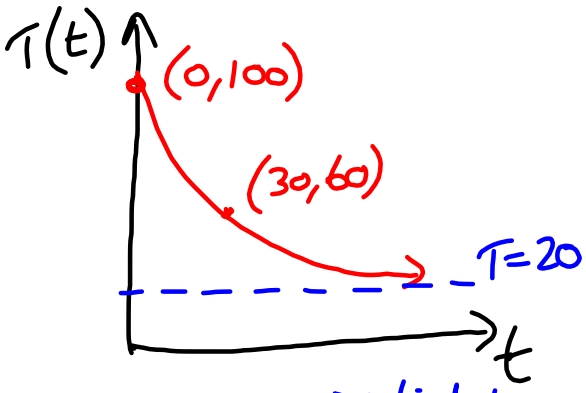
7. A cup of hot liquid was left to cool in a room whose temperature was 20 °C.
C The temperature changes with time according to the function $T(t) = 80(\frac{1}{2})^{\frac{t}{30}} + 20$. Use your knowledge of transformations to sketch this function. Explain the meaning of the y-intercept and the asymptote in the context of this problem.

y-int is $T(0)$
$= 80(\frac{1}{2})^{0/30} + 20$
$= 80(1) + 20$
$= 100$

Transform point $(1, \frac{1}{2})$
$1 \to \frac{1}{1/30} = 30$
$\frac{1}{2} \to 80(\frac{1}{2}) + 20 = 60$

Anchor points are
(0,100) and (30, 60)



y-intercept = initial temp
asymptote = room temp
[can't be lower than this]

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9. Match the equation of the functions from the list to the appropriate graph at the top of the next page.

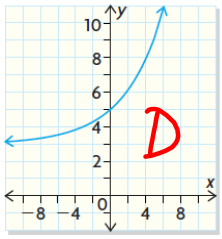
a) $f(x) = -\left(\frac{1}{4}\right)^{-x} + 3$

b) $y = \left(\frac{1}{4}\right)^{x} + 3$

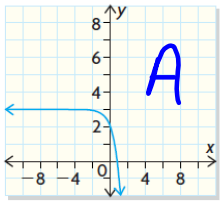
c) $g(x) = -\left(\frac{5}{4}\right)^{-x} + 3$

d) $h(x) = 2\left(\frac{5}{4}\right)^{x} + 3$

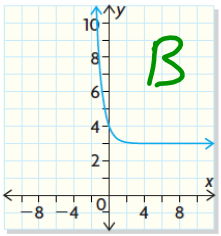
i)



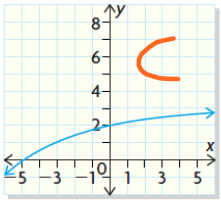
iii)



ii)



iv)



y-int = 5

Initially a decay that has been reflected in both axes Base is further from one so steeper.

y-int = 4

Initially a growth that has been reflected in both axes. Base is closer to one, so less steep.

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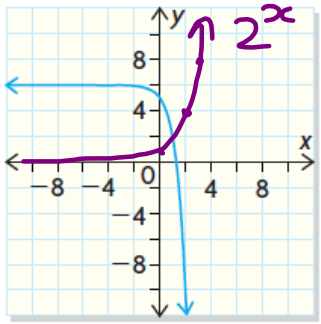
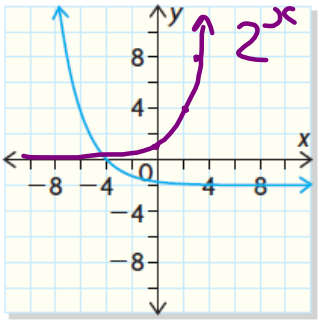
10. Each graph represents a transformation of the function $f(x) = 2^x$. Write an equation for each one.

a)



b)



Asymptote $y = 6$

Base is 2^x

This has been reflected in x-axis

$\Rightarrow$ a is negative

Asymptote $y = -2$

Base is 2^x

This has been reflected in the y-axis $\Rightarrow$ k is neg

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