Solutions

1. Calculate the percentile ~~rank~~ for each student's quiz result in a grade 12 math class.

Mark	Frequency
4.0	2
5.0	6
6.0	8
7.0	13
8.0	4
9.0	3
10.0	2

Total the frequency = 38

The 1st ranked mark is 4.0.
No values less than this so L = 0.
There two values of 4.0 so E = 2.

$$p = 100\frac{(L+0.5E)}{n}$$
$$= 100\frac{0+0.5(2)}{38}$$
$$\approx 2.6$$

A mark of 4.0 is in the 3rd percentile.

Using the formula, repeat the percentile rank calculation for the other marks.

Mark	Frequency	Percentile ~~Rank~~
4.0	2	$100\frac{0+0.5(2)}{38} \approx 3$
5.0	6	$100\frac{2+0.5(6)}{38} \approx 13$
6.0	8	$100\frac{8+0.5(8)}{38} \approx 32$
7.0	13	$100\frac{16+0.5(13)}{38} \approx 59$
8.0	4	$100\frac{29+0.5(4)}{38} \approx 82$
9.0	3	$100\frac{33+0.5(3)}{38} \approx 91$
10.0	2	$100\frac{36+0.5(2)}{38} \approx 97$

2. What is the range of the data set?

23 56 45 65 59
55 62 54 85 25

Range = Highest - Lowest

= 85 - 23

= 62

3. Calculate the median, range, Q1, Q3, and interquartile range of each set of data. Identify any outliers.

a) 39 51 35 22 28 67 57
42 56 74 51 87 99 48
36 28 57 23 53 74

Number	Raw Data	Ordered
1	39	22
2	42	23
3	36	28
4	51	28
5	56	35
6	28	36
7	35	39
8	74	42
9	57	48
10	22	51
11	51	51
12	23	53
13	28	56
14	87	57
15	53	57
16	67	67
17	99	74
18	74	74
19	57	87
20	48	99

First, sort the 20 data points (by hand or tech)

Median is midway between 10th and 11th values = 51

Q1 is midway between 5th and 6th values = 35.5

Q3 is midway between 15th and 16th values = 62

Range = Highest - Lowest

= 99 - 22

= 77

Interquartile Range (IQR) = Q3 - Q1

= 62 - 35.5

= 26.5

Outliers are defined as any values that are more than 1.5 times the IQR below Q1 or above Q3

Lower Limit:

= Q1 - 1.5(IQR)

= 35.5 - 1.5(26.5)

= -4.25

Upper Limit:

= Q3 + 1.5(IQR)

= 62 + 1.5(26.5)

= 101.75

All of the data points lie within these limits so there are no outliers (despite what it says in the back of the book).

3. Calculate the median, range, Q1, Q3, and interquartile range of each set of data. Identify any outliers.

b) 245 264 222 213 243
215 467 264 276 199
127 216 233

Number	Raw Data	Ordered
1	245	127
2	215	199
3	127	213
4	264	215
5	467	216
6	216	222
7	222	233
8	264	243
9	233	245
10	213	264
11	276	264
12	243	276
13	199	467

First, sort the 13 data points (by hand or tech)

Median is 7th value = 233

Q1 is midpoint of 3rd and 4th values = 214

Q3 is midpoint of 10th and 11th values = 264

Range = Highest - Lowest
= 467 - 127
= 340

Interquartile Range (IQR) = Q3 - Q1
= 264 - 214
= 50

Outliers are defined as any values that are more than 1.5 times the IQR below Q1 or above Q3

Lower Limit:
= Q1 - 1.5(IQR)
= 214 - 1.5(50)
= 139

Upper Limit:
= Q3 + 1.5(IQR)
= 264 + 1.5(50)
= 339

There are two data points that lies outside these limits (127 & 467) so these are outliers.

3. Calculate the median, range, Q1, Q3, and interquartile range of each set of data. Identify any outliers.

c) 5 7 9 4 5 7 3 2 6
2 8 5 9 1 3 3 0 4
6 8 5 9 2 3 4 5 0
4 2 4 6 2 5 6

Number	Raw Data	Ordered
1	5	0
2	2	0
3	6	1
4	4	2
5	7	2
6	8	2
7	8	2
8	2	3
9	9	3
10	5	3
11	5	3
12	4	3
13	4	4
14	9	4
15	9	4
16	6	4
17	5	5
18	1	5
19	2	5
20	2	5
21	7	5
22	3	5
23	3	5
24	5	6
25	3	6
26	3	6
27	4	6
28	6	7
29	2	7
30	0	8
31	5	8
32	6	9
33	4	9
34	0	9

First, sort the 34 data points (by hand or tech)

Median is midway between 17th and 18th values = 4.5

Q1 is midpoint of 8th and 9th values = 2.5

Q3 is midpoint of 26th and 27th values = 6

Range = Highest - Lowest
= 9 - 0
= 9

Interquartile Range (IQR) = Q3 - Q1
= 6 - 2.5
= 3.5

Outliers are defined as any values that are more than 1.5 times the IQR below Q1 or above Q3

Lower Limit:
= Q1 - 1.5(IQR)
= 3 - 1.5(3.5)
= -2.25

Upper Limit:
= Q3 + 1.5(IQR)
= 6 + 1.5(3.5)
= 11.25

All of the data points lie within these limits so there are no outliers.

3. Calculate the median, range, Q1, Q3, and interquartile range of each set of data. Identify any outliers.

d) 6213 7985 3426 5134
7659 3462 5348 6213
5968 7659 3567

Number	Raw Data	Ordered
1	6213	3426
2	7659	3462
3	5968	3567
4	7985	5134
5	3462	5348
6	7659	5968
7	3426	6213
8	5348	6213
9	3567	7659
10	5134	7659
11	6213	7985

First, sort the 11 data points (by hand or tech)

Median is 6th value = 5968

Q1 is 3rd value = 3567

Q3 is 9th value = 7659

Range = Highest - Lowest
= 7985 - 3426
= 4559

Interquartile Range (IQR) = Q3 - Q1
= 7659 - 3567
= 4092

Outliers are defined as any values that are more than 1.5 times the IQR below Q1 or above Q3

Lower Limit:
= Q1 - 1.5(IQR)
= 3567 - 1.5(4092)
= -2571

Upper Limit:
= Q3 + 1.5(IQR)
= 7659 + 1.5(4092)
= 13797

All of the data points lie within these limits so there are no outliers.

4. If each number in a set is increased by two, which of the measures of spread would remain unchanged?

A the range
B the interquartile range
C the percentiles
D all of the above

If every value is increased by two, the order of the new values will be the same compared to the old values, so the percentiles will stay the same as will the range and IQR. ⟶ D

5. Which is an incorrect statement about the interquartile range?

A It contains the middle 50% of the data.
B An outlier lies more than 1.5 times the IQR from Q1 or Q3.
C To calculate the interquartile range, subtract Q3 – Q1.
D The median always lies at the middle of the interquartile range.

The median does not ALWAYS lie in the middle of the IQR. The other statements are true. ⟶ D

6. a) Rank the provinces in ascending order by their 2011 infant mortality rates.

b) Determine the percentile ranks for five provinces or territories of your choice.

a)

Rank	Infant Mortality Rate by Prov/Ter	2011
1	Yukon	0.0
2	New Brunswick	3.5
3	British Columbia	3.8
4	PEI	4.2
5	Quebec	4.3
6	Ontario	4.6
7	Nova Scotia	4.9
8	Alberta	5.3
9	Newfoundland & Labrador	6.3
10	Saskatchewan	6.7
11	NW Territories	7.2
12	Manitoba	7.7
13	Nunavut	26.3

The infant mortality rate represents the number of children, per thousand, who die before the age of one year.

Infant Mortality Rates by Province and Territory

	2007	2008	2009	2010	2011
Newfoundland and Labrador	7.5	5.1	6.3	5.3	6.3
Prince Edward Island	5.0	2.0	3.4	3.6	4.2
Nova Scotia	3.3	3.5	3.4	4.6	4.9
New Brunswick	4.3	3.2	5.8	3.4	3.5
Québec	4.5	4.3	4.4	5.0	4.3
Ontario	5.2	5.3	5.0	5.0	4.6
Manitoba	7.3	6.5	6.3	6.7	7.7
Saskatchewan	5.8	6.2	6.7	5.9	6.7
Alberta	6.0	6.2	5.5	5.9	5.3
British Columbia	4.0	3.7	3.6	3.8	3.8
Yukon	8.5	5.4	7.8	5.2	0.0
Northwest Territories	4.1	9.7	15.5	1.4	7.2
Nunavut	15.1	16.1	14.8	14.5	26.3
Canada	5.1	5.1	4.9	5.0	4.8

Source: Infant mortality rates, by province and territory (both sexes), Statistics Canada.

b) Here are the solutions for:

New Brunswick **i)** A rate of 3.5 is the 2nd ranked rate. There is 1 data value less than 3.5, so $L = 1$. There is 1 data value equal to 9.2, so $E = 1$.

$$p = 100\frac{(L + 0.5E)}{n}$$

$$= 100\frac{1 + 0.5(1)}{13}$$

$$\approx 12$$

An infant mortality rate of 3.5 is in the 12th percentile.

PEI **ii)** A rate of 4.2 is the 4th ranked rate. There are 3 data values less than 4.2, so $L = 3$. There is 1 data value equal to 4.2, so $E = 1$.

$$p = 100\frac{(L + 0.5E)}{n}$$

$$= 100\frac{3 + 0.5(1)}{13}$$

$$\approx 27$$

An infant mortality rate of 4.2 is in the 27th percentile.

Ontario **iii)** A rate of 4.6 is the 6th ranked rate. There are 5 data values less than 4.6, so $L = 5$. There is 1 data value equal to 4.6, so $E = 1$.

$$p = 100\frac{(L + 0.5E)}{n}$$

$$= 100\frac{5 + 0.5(1)}{13}$$

$$\approx 42$$

An infant mortality rate of 4.6 is in the 42th percentile.

Alberta **iv)** A rate of 5.3 is the 8th ranked rate. There are 7 data values less than 5.3, so $L = 7$. There is 1 data value equal to 5.3, so $E = 1$.

$$p = 100\frac{(L + 0.5E)}{n}$$
$$= 100\frac{7 + 0.5(1)}{13}$$
$$\approx 58$$

An infant mortality rate of 5.3 is in the 58th percentile.

and NW Territories

v) A rate of 7.2 is the 11th ranked rate. There are 10 data values less than 7.2, so $L = 10$. There is 1 data value equal to 7.2, so $E = 1$.

$$p = 100\frac{(L + 0.5E)}{n}$$
$$= 100\frac{10 + 0.5(1)}{13}$$
$$\approx 81$$

An infant mortality rate of 7.2 is in the 81st percentile.

7. a) Determine the median and interquartile range for the infant mortality rate in each year.

b) Compare these measures across the years.

c) Why would the medians not be the same as the mortality rate for all of Canada?

The infant mortality rate represents the number of children, per thousand, who die before the age of one year.

Infant Mortality Rates by Province and Territory

	2007	2008	2009	2010	2011
Newfoundland and Labrador	7.5	5.1	6.3	5.3	6.3
Prince Edward Island	5.0	2.0	3.4	3.6	4.2
Nova Scotia	3.3	3.5	3.4	4.6	4.9
New Brunswick	4.3	3.2	5.8	3.4	3.5
Québec	4.5	4.3	4.4	5.0	4.3
Ontario	5.2	5.3	5.0	5.0	4.6
Manitoba	7.3	6.5	6.3	6.7	7.7
Saskatchewan	5.8	6.2	6.7	5.9	6.7
Alberta	6.0	6.2	5.5	5.9	5.3
British Columbia	4.0	3.7	3.6	3.8	3.8
Yukon	8.5	5.4	7.8	5.2	0.0
Northwest Territories	4.1	9.7	15.5	1.4	7.2
Nunavut	15.1	16.1	14.8	14.5	26.3
Canada	5.1	5.1	4.9	5.0	4.8

Source: Infant mortality rates, by province and territory (both sexes), Statistics Canada.

a)

	2007	2008	2009	2010	2011
Median =median(highlighted data)	5.2	5.3	5.8	5.0	4.9
Q1	4.2	3.6	4	3.7	4
Q3	7.4	6.35	7.25	5.9	6.95
IQR	3.2	2.75	3.25	2.2	2.95

If you type the data into Excel you can then get it to perform the calculations for you. In an empty cell type in the red info and highlight the data you wish to use where it says highlighted data. For this set of data Excel wasn't calculating the quartiles correctly so they had to be calculated manually.

b) The median infant mortality rate increased from 2007 to 2009 and then decreased through to 2011. The IQR went down, up, down, and up for the given period under review.

c) The median may not be the same as the infant mortality rate because of possible outliers.

8. a) Are the 2011 mortality rates for Yukon and Nunavut outliers?

b) Explain the variability of the mortality rates in Yukon, Northwest Territories, and Nunavut as compared to Ontario.

Rank	Infant Mortality Rate by Prov/Ter	2011
1	Yukon	0.0
2	New Brunswick	3.5
3	British Columbia	3.8
4	PEI	4.2
5	Quebec	4.3
6	Ontario	4.6
7	Nova Scotia	4.9
8	Alberta	5.3
9	Newfoundland & Labrador	6.3
10	Saskatchewan	6.7
11	NW Territories	7.2
12	Manitoba	7.7
13	Nunavut	26.3

Outliers are defined as any values that are more than 1.5 times
the IQR below Q1 or above Q3

Lower Limit:

= Q1 - 1.5(IQR)

= 4 - 1.5(2.95)

= -0.425

Upper Limit:

= Q3 + 1.5(IQR)

= 6.95 + 1.5(2.95)

= 11.375

The infant mortality rate represents the number of children, per thousand, who die before the age of one year.

Infant Mortality Rates by Province and Territory

	2007	2008	2009	2010	2011
Newfoundland and Labrador	7.5	5.1	6.3	5.3	6.3
Prince Edward Island	5.0	2.0	3.4	3.6	4.2
Nova Scotia	3.3	3.5	3.4	4.6	4.9
New Brunswick	4.3	3.2	5.8	3.4	3.5
Québec	4.5	4.3	4.4	5.0	4.3
Ontario	5.2	5.3	5.0	5.0	4.6
Manitoba	7.3	6.5	6.3	6.7	7.7
Saskatchewan	5.8	6.2	6.7	5.9	6.7
Alberta	6.0	6.2	5.5	5.9	5.3
British Columbia	4.0	3.7	3.6	3.8	3.8
Yukon	8.5	5.4	7.8	5.2	0.0
Northwest Territories	4.1	9.7	15.5	1.4	7.2
Nunavut	15.1	16.1	14.8	14.5	26.3
Canada	5.1	5.1	4.9	5.0	4.8

Source: Infant mortality rates, by province and territory (both sexes), Statistics Canada.

The Yukon (0.0) is inside the lower limit, but Nunavut (26.3) is outside the upper limit so Nunavut is an outlier.

b) There could be any number of reasons why these vary so much from the data for Ontario. The Yukon could be very low (0.0) due to a very low number of births that year... NW Territories and Nunavut may have fewer hospitals to care for sick children, or their populations could be prone to disease.