

Solutions

1. Determine the mean, median, and mode for each set of data.

a) 4 6 9 12 15 7 13 4 7 10 3 8 15

$$\begin{aligned}\mu &= \frac{\sum x}{n} \\ &= \frac{4+6+9+12+15+7+13+4+7+10+3+8+15}{13} \\ &= \frac{113}{13} \\ &\approx 8.7\end{aligned}$$

The mean is 8.7, the median is 8, and the modes are 4, 7, and 15.

For the median, order the data and locate the middle value.

3 4 4 6 7 7 8 9 10 12 13 15 15

The mode appears the most often: 4, 7, and 15.

1. Determine the mean, median, and mode for each set of data.

b) 9 8 20 23 12 12 9 9 12 9 20 21 9

$$\begin{aligned}\mu &= \frac{\sum x}{n} \\ &= \frac{9 + 8 + 20 + 23 + 12 + 12 + 9 + 9 + 12 + 9 + 20 + 21 + 9}{13} \\ &= \frac{173}{13} \\ &\approx 13.3\end{aligned}$$

The mean is 13.3, the median is 12, and the mode is 9.

For the median, order the data and locate the middle value.

8 9 9 9 9 7 12 12 12 20 20 21 23

The mode appears the most often: 9.

1. Determine the mean, median, and mode for each set of data.

c) 110 152 112 124 110 134 138 127 118 110
114 162

$$\begin{aligned}\mu &= \frac{\sum x}{n} \\ &= \frac{110 + 152 + 112 + 124 + 110 + 134 + 138 + 127 + 118 + 110 + 114 + 162}{12} \\ &= \frac{1511}{12} \\ &\approx 125.9\end{aligned}$$

For the median, order the data and locate the two middle values and determine the average.

110 110 110 112 114 118 124 127 134 138 152 162

$$\frac{118 + 124}{2} = 121$$

The mode appears the most often: 110.

The mean is 125.9, the median is 121, and the mode is 110.

2. Nina runs the 400-m race for Mustang High School. Her times in the last six track meets were 1.45 min, 1.50 min, 1.42 min, 1.41 min, 1.42 min, and 1.48 min.

- a) What are the mean and median for her running times?
 b) Which measure of central tendency best describes Nina's average time? Explain.

$$\begin{aligned} \text{a) } \mu &= \frac{\sum x}{n} \\ &= \frac{1.45 + 1.50 + 1.42 + 1.41 + 1.42 + 1.48}{6} \\ &= \frac{8.68}{6} \\ &\approx 1.45 \end{aligned}$$

The mean is 1.45 minutes and the median is 1.44 minutes.

For the median, order the data and locate the two middle values and determine the average.

1.41 1.42 1.42 1.45 1.48 1.50

$$\frac{1.42 + 1.45}{2} = 1.435$$

- b) The median is the best measure to describe Nina's average. Half of her times are above it and half are below.

3. The observation that occurs most frequently in a data sample is the

- A mean
 B weighted mean
 C mode
 D median

C - MODE is MOST often occurring value(s). There can be no mode if all of them appear only once.

4. What is the median of the sample 5, 5, 11, 9, 8, 5, 8?

- A 9
 B 6
 C 5
 D 8

D Seven values, we need the middle one (4th) once they have been ordered.

5. The mean of Daniel's marks on five tests was 77.4. His marks on the first four tests were 88, 77, 70, and 72. Calculate Daniel's mark on the fifth test.

Use the formula for the mean and solve for the missing value, x .

$$\mu = \frac{\sum x}{n}$$

$$77.4 = \frac{88 + 77 + 70 + 72 + x}{5}$$

$$387 = 307 + x$$

$$80 = x$$

Daniel's fifth mark was 80.

7. **Communication** Determine whether the argument is valid for each situation. Explain your thinking.
- a) An advertising company has a mean monthly sales record of \$16 235. Therefore, half the team members sold more than \$16 235.
- b) A survey shows that 78% of all salaries are below the mean. Therefore, there must be a mistake.
- c) The mean mark of one class is 71, while the mean mark of another class is 76. Therefore, the mean of the two classes is 73.5.
- d) My median monthly expenses total \$850. Therefore, my total expenses for the year must be \$10 200.

a) Invalid. The mean isn't necessarily in the middle of the data, so the company is incorrect to claim this.

b) Invalid. The mean can range of positions within a data set. Outliers can cause this to happen eg. A CEO having a huge salary will cause an increase in the mean wage.

c) The argument could be valid, if the class sizes are the same, or invalid if the class sizes are different. By definition, the mean of the two classes is

$$\frac{\text{sum of marks in class A} + \text{sum of marks in class B}}{\text{number in class A} + \text{number in class B}}$$

Then the mean of two means is

$$0.5 \left(\frac{\text{sum of marks in class A}}{\text{number in class A}} + \frac{\text{sum of marks in class B}}{\text{number in class B}} \right)$$

$$= 0.5 \left(\frac{(\text{number in class B})(\text{sum of marks in class A}) + (\text{number in class A})(\text{sum of marks in class B})}{\text{number in class A} + \text{number in class B}} \right)$$

d) Invalid. The median multiplied by 12 does not equal the total for the year... The mean multiplied by 12 does.

8. **Communication** Which measure of central tendency would be best suited for each situation? Explain why you chose the measure that you did.
- a) a summary of a class's report card marks
- b) an award for the most popular movie of the year
- c) an employer budgeting for the average salary of its employees
- d) a potential employee looking for the typical salary among current employees

a) The **MEDIAN** would be most appropriate. The mode could be anywhere and one or two very low (or high) marks can affect the mean on a small sample size like the number of students in a class.

b) The **MODE** would be most appropriate because you are looking for the winner of a popularity vote: the movie that gets the most.

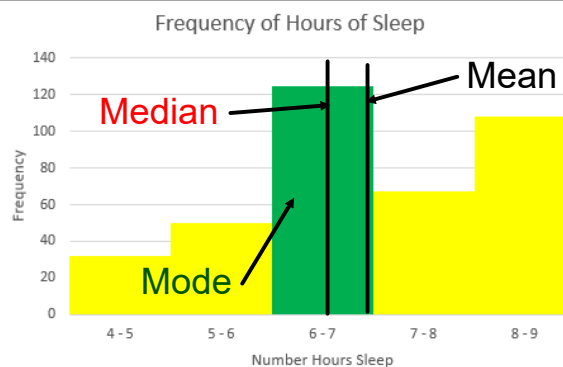
c) The **MEAN** would be the most appropriate because it is the total budget divided by the number of staff.

d) The **MEDIAN** would be the most appropriate because as previously stated the mode could be anywhere and the mean can be distorted by one or two very low or high salaries.

10. Michael surveyed the grade 12 students at his school to research the number of hours of sleep they got. He asked them how many hours of sleep they got last night. The table shows his results.

| Time (h) | 4-5 | 5-6 | 6-7 | 7-8 | 8-9 |
|-----------|-----|-----|-----|-----|-----|
| Frequency | 32 | 50 | 125 | 67 | 108 |

- a) Make a histogram of these data.
- b) Estimate the mean, median, and modal interval for the hours of sleep by grade 12 students.
- c) Mark the measures of central tendency on your histogram.
- d) Discuss any skewing of the data and how it relates to the measures of central tendency.



- b) **Mode = 6-7 hours (tallest column)**

$$\text{Median} = (382+1)/2 \text{ position}$$

$$= 191.5, \text{ so between } 191 \text{ and } 192.$$

This occurs in the grouping 6-7 hours (83rd to 207th person) so the median = **6.5 hours** (midpoint).

$$\text{Grouped mean} = 6.94\dots$$

| Time (h) | Midpoint | Frequency | Midpoint x Frequency | Cumulative Frequency |
|----------|----------|-----------------------|----------------------|----------------------|
| 4 - 5 | 4.5 | 32 | 144 | 32 |
| 5 - 6 | 5.5 | 50 | 275 | 82 |
| 6 - 7 | 6.5 | 125 | 812.5 | 207 |
| 7 - 8 | 7.5 | 67 | 502.5 | 274 |
| 8 - 9 | 8.5 | 108 | 918 | 382 |
| | | Totals | 2652 | 382 |
| | | Grouped Mean = | 6.942408377 | |

d) The data is negatively skewed so the modal interval is the least appropriate. The mean and median are similar in value so you could use either of those.

13. Karen's term mark is 82%. The term counts for 70% of the final mark. What mark must Karen achieve on the exam to earn a final mark of
- a) 80%? b) 85%?
 c) at least 75%?
 d) Can Karen achieve 88%? Explain.

- a) The final exam is worth 30%. Determine the final exam score, E , needed for Karen to receive a final mark of 80% in the course.

$$\mu = \frac{\sum x_i w_i}{\sum w_i}$$

$$80 = \frac{82(0.70) + E(0.30)}{1.0}$$

$$80 = 57.4 + 0.30E$$

$$22.6 = 0.30E$$

$$75.33 = E$$

Karen needs an exam mark of 76% to achieve a final course mark of 80%.

- b) The final exam is worth 30%. Determine the final exam score, E , needed for Karen to receive a final mark of 85% in the course.

$$\mu = \frac{\sum x_i w_i}{\sum w_i}$$

$$85 = \frac{82(0.70) + E(0.30)}{1.0}$$

$$85 = 57.4 + 0.30E$$

$$27.6 = 0.30E$$

$$92 = E$$

Karen needs an exam mark of 92% to achieve a final course mark of 80%.

13. Karen's term mark is 82%. The term counts for 70% of the final mark. What mark must Karen achieve on the exam to earn a final mark of
- a) 80%? b) 85%?
 c) at least 75%?
 d) Can Karen achieve 88%? Explain.

- c) The final exam is worth 30%. Determine the final exam score, E , needed for Karen to receive a final mark of at least 75% in the course.

$$\mu = \frac{\sum x_i w_i}{\sum w_i}$$

$$75 = \frac{82(0.70) + E(0.30)}{1.0}$$

$$75 = 57.4 + 0.30E$$

$$17.6 = 0.30E$$

$$58.67 = E$$

Karen needs an exam mark of 59% to achieve a final course mark of 80%.

- d) The final exam is worth 30%. Determine the final exam score, E , needed for Karen to receive a final mark of at least 88% in the course.

$$\mu = \frac{\sum x_i w_i}{\sum w_i}$$

$$88 = \frac{82(0.70) + E(0.30)}{1.0}$$

$$88 = 57.4 + 0.30E$$

$$30.6 = 0.30E$$

$$102 = E$$

Karen needs an exam mark of 102% to achieve a final course mark of 88%. She cannot achieve this unless there are bonus mark available.