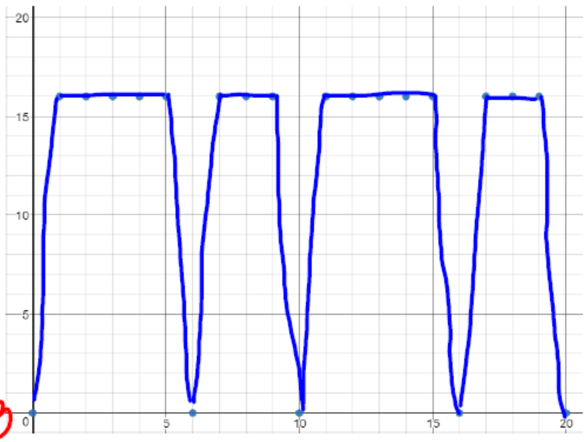
Solutions

1. The automatic dishwasher in a school cafeteria runs constantly through lunch. The table shows the amount of water in the dishwasher at different times.

Time (min)	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Volume (L)	0	16	16	16	16	16	0	16	16	16	0	16	16	16	16	16	0	16	16	16	0

a) Plot the data, and draw the resulting graph.
b) Is the graph periodic?
c) What is the period of the function, and what does it represent in this situation?
d) Determine the equation of the axis.
e) Determine the amplitude.
f) What is the range of this function?



b) Yes

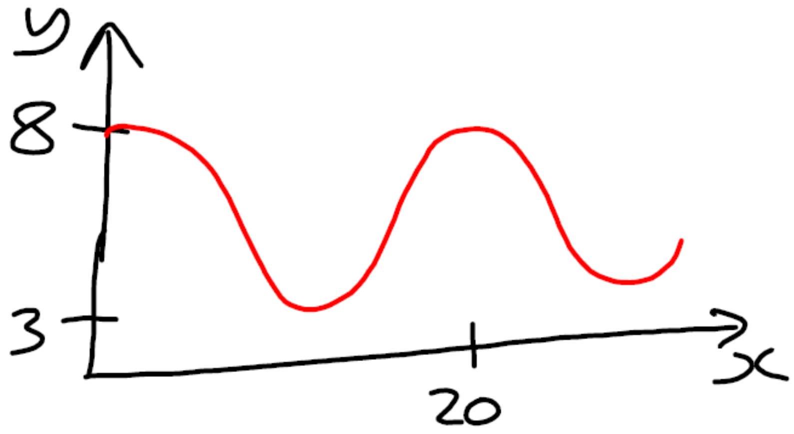
c) 10 minutes
Length of time cleaning and drying

d) Axis $= \frac{16+0}{2} = 8 \Rightarrow V = 8$

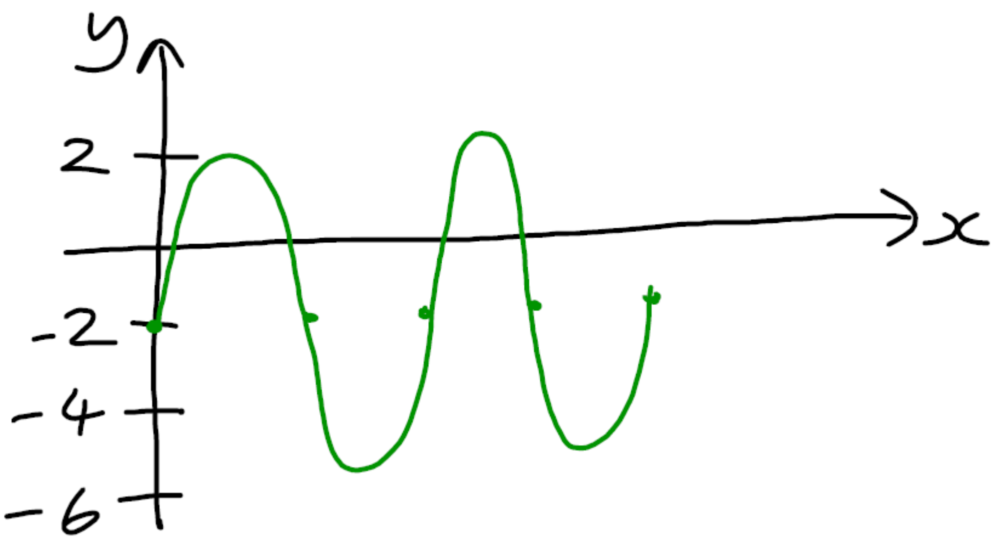
e) Amplitude $= \frac{16-0}{2} = 8$

f) $R = \{V \in \mathbb{R} \mid 0 \leq V \leq 16\}$

2. Sketch a graph of a periodic function whose period is 20 and whose range is $\{y \in \mathbf{R} \mid 3 \le y \le 8\}$.



3. Sketch the graph of a sinusoidal function that has a period of 6, an amplitude of 4, and whose equation of the axis is $y = -2$.



4. Colin is on a unique Ferris wheel: it is situated on the top of a building. Colin's height above the ground at various times is recorded in the table.

Time (s)	0	10	20	30	40	50	60	70	80	90	100	110	120	130	140	150	160
Height (m)	25	22.4	16	9.7	7	9.7	16	22.4	25	22.4	16	9.7	7	9.7	16	22.4	25

a) What is the period of the function, and what does it represent in this situation?
b) What is the equation of the axis, and what does it represent in this situation?
c) What is the amplitude of the function, and what does it represent in this situation?
d) Was the Ferris wheel already in motion when the data were recorded? Explain.
e) How fast is Colin travelling around the wheel, in metres per second?
f) What is the range of the function?
g) If the building is 6 m tall, what was Colin's boarding height in terms of the building?

a) Period = 80 seconds
Time taken for one revolution

b) Axis $= \frac{25+7}{2} = 16$
$\Rightarrow h = 16$

c) Amplitude $= \frac{25-7}{2} = 9$ Radius of the wheel

d) Yes. Colin starts at a maximum. You usually get on at a minimum.

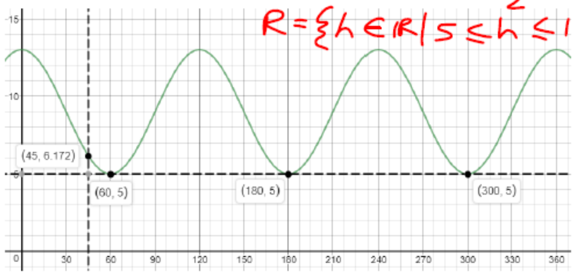
e) Speed $= \frac{\text{Distance}}{\text{Time}} = \frac{2\pi r}{\text{period}} = \frac{2 \times 3.14 \times 9}{80}$
$= 0.71$ m/s [This is called ANGULAR VELOCITY and is a G12 topic]

f) $R = \{h \in \mathbb{R} \mid 7 \le h \le 25\}$

g) Boarding height = max − height of building
$= 25 - 6$
$= 19$m

5. a) Graph the function $h(x) = 4\cos(3x) + 9$ using a graphing calculator in DEGREE mode for $0° \le x \le 360°$. Use Xscl = 90°. Determine the period, equation of the axis, amplitude, and the range of the function.
b) Is the function sinusoidal?
c) Calculate $h(45)$.
d) Determine the values of x, $0° \le x \le 360°$, for which $h(x) = 5$.

a) Period $= 120°$
Eqn of axis $= \frac{13+5}{2}$
$h = 9$
Amplitude $= \frac{13-5}{2} = 4$
$R = \{h \in \mathbb{R} \mid 5 \le h \le 13\}$



b) Yes.

c) $h(45) = 4\cos(3(45)) + 9$
$= 6.17$m

Period $= \frac{360}{3} = 120°$

d) $5 = 4\cos(3x) + 9$
$\frac{-4}{4} = \frac{4\cos(3x)}{4}$
$-1 = \cos(3x)$
$\cos^{-1}(-1) = 3x$
$\frac{180}{3} = \frac{3x}{3}$
$\underline{x = 60°}$

Special case that has only one solution

$\Rightarrow x = 60 + 120$
$\underline{x = 180°}$
$\Rightarrow x = 180 + 120$
$\underline{x = 300°}$

6. A ship is docked in port and rises and falls with the waves. The function $d(t) = 2\sin(30t)^\circ + 5$ models the depth of the propeller, $d(t)$, in metres at t seconds. Graph the function using a graphing calculator, and answer the following questions.

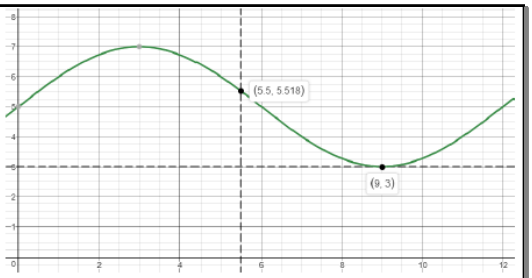
a) What is the period of the function, and what does it represent in this situation?

b) If there were no waves, what would be the depth of the propeller?

c) What is the depth of the propeller at $t = 5.5$ s?

d) What is the range of the function?

e) Within the first 10 s, at what times is the propeller at a depth of 3 m?



a) period $= \frac{360}{k} = \frac{360}{30} = 12$ seconds

Time taken for propeller to rise and fall.

b) $\Rightarrow$ axis of curve, $d = 5$

c) $d(5.5) = 2\sin(30(5.5)) + 5$

$= 5.5$m

d) $R = \{d \in \mathbb{R} \mid 3 \leq d \leq 7\}$

$\begin{bmatrix} \max = c + |a| \\ \min = c - |a| \end{bmatrix}$

e) $3 = 2\sin(30t) + 5$

$\frac{-2}{2} = \frac{2\sin(30t)}{2}$

$-1 = \sin(30t)$

$\sin^{-1}(-1) = 30t$

Special case - one solution

$\frac{270}{30} = \frac{30t}{30}$

$t = 9$ seconds

8. Each sinusoidal function has undergone one transformation that may have affected the period, amplitude, or equation of the axis of the function. In each case, determine which characteristic has been changed. If one has, indicate its new value.

a) $y = \sin x - 3$

b) $y = \sin(4x)$

c) $y = 7\cos x$

d) $y = \cos(x - 70^\circ)$

a) Axis of curve $y = -3$ $[y = c]$

b) Period now 90° $\left[\frac{360}{k}\right]$

c) Amplitude now 7 $[a]$

d) No change. Only has a phase shift of right 70°.

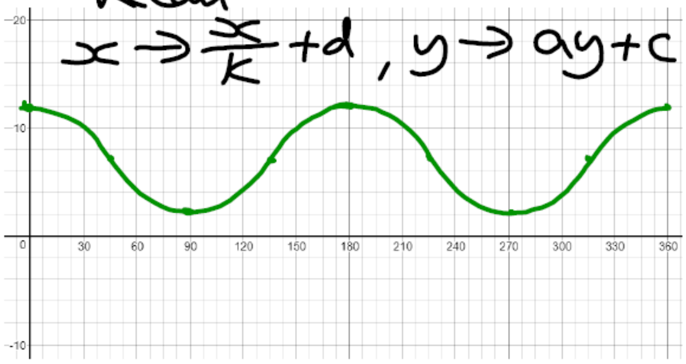
9. Use transformations to graph each function for $0° \leq x \leq 360°$.

a) $y = 5\cos(2x) + 7$

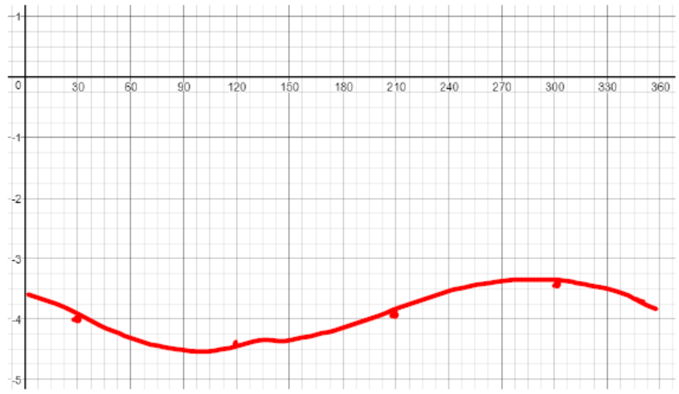
b) $y = -0.5\sin(x - 30°) - 4$

Recall: $x \to \frac{x}{k} + d$, $y \to ay + c$

x	y	new x	new y
0	1	0	12
90	0	45	7
180	-1	90	2
270	0	135	7
360	1	180	12



x	y	new x	new y
0	0	30	-4
90	1	120	$-4\frac{1}{2}$
180	0	210	-4
270	-1	300	$-3\frac{1}{2}$
360	0	390	-4



10. Determine the range of each sinusoidal function without graphing.

a) $y = -3\sin(4x) + 2$

b) $y = 0.5\cos(3(x - 40°))$

Recall: $Max = c + |a|$
$Min = c - |a|$

a) $Max = 2 + |-3| = 5$ $\quad Min = 2 - |-3| = -1$

$\Rightarrow R = \{y \in \mathbb{R} \mid -1 \leq y \leq 5\}$

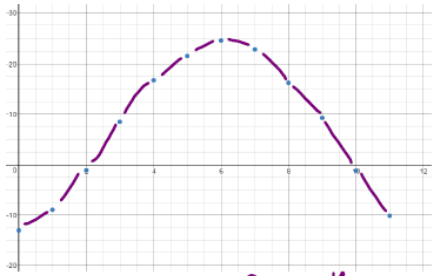
b) $Max = 0 + |0.5| = 0.5$ $\quad Min = 0 - |0.5| = -0.5$

$\Rightarrow R = \{y \in \mathbb{R} \mid -0.5 \leq y \leq 0.5\}$

11. The average daily maximum temperature in Kenora, Ontario, is shown for each month.

Time (months)	J	F	M	A	M	J	J	A	S	O	N	D
Temperature (°C)	−13.1	−9.0	−1.1	8.5	16.8	21.6	24.7	22.9	16.3	9.3	−1.2	−10.2

a) Prepare a scatter plot of the data. Let January represent month 0.
b) Draw a curve of good fit. Explain why this type of data can be expressed as a periodic function.
c) State the maximum and minimum values.
d) What is the period of the curve? Explain why this period is appropriate within the context of the question.
e) Write an equation for the axis of the curve.
f) What is the phase shift if the cosine function acts as the base curve?
g) Use the cosine function to write an equation that models the data.
h) Use the equation to predict the temperature for month 38. How can the table be used to confirm this prediction?



b) Follows a repeating pattern over 12 months

c) Max = 24.7°C Min = −13.1°C

d) 12 months. Temperature cycle repeats every year

e) Axis $= \frac{24.7+(-13.1)}{2} = 5.8 \Rightarrow T = 5.8$

f) Cosine starts at a max $\Rightarrow$ phase shift right 6

g) $T(t) = \underbrace{18.9}_{\substack{\text{amplitude} \\ \frac{\text{max}-\text{min}}{2}}}\cos(\underbrace{30}_{\frac{360}{\text{period}}}(x - \underbrace{6}_{\text{phase shift}})) + \underbrace{5.8}_{\text{axis of curve}}$

h) $T(38) = 18.9\cos(30(38-6)) + 5.8$

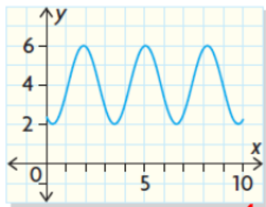
$= -3.65$°C

Month 38 represents March (−1.1°C) so the value is reasonable.

12. Determine the sine function $y = a \sin k(\theta - d) + c$ for each graph.

a)

b)



Max = 3.5 Min = 1.5

$\Rightarrow$ Axis $(c) = \frac{3.5+1.5}{2}$

$= 2.5$

$\Rightarrow$ Amplitude $(a) = \frac{3.5-1.5}{2}$

$= 1$

Period $= 2(6)$

$= 12$

$\Rightarrow k = \frac{360}{12} = 30$

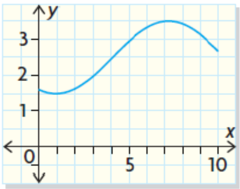
If using −cosine $\Rightarrow$ starts at a min

so phase shift (d) is right 1

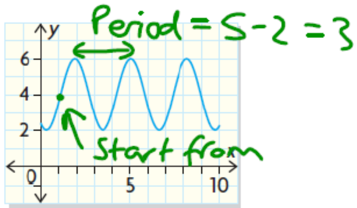
$\Rightarrow -\cos(30(\theta - 1)) + 2.5$

12. Determine the sine function $y = a \sin k(\theta - d) + c$ for each graph.

a)



b)



Max = 6 Min = 2

$\Rightarrow$ Axis (c) $= \dfrac{6+2}{2}$
$= 4$

$\Rightarrow$ Amplitude (a) $= \dfrac{6-2}{2}$
$= 2$

Period = 3

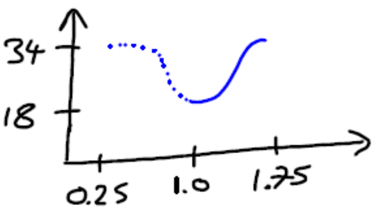
$\Rightarrow k = \dfrac{360}{3} = 120$

Start increasing from the axis
$\Rightarrow$ use positive sine with phase shift (d) of right 1.

$\Rightarrow y = 2\sin(120(\theta - 1)) + 4$

13. Meagan is sitting in a rocking chair. The distance, $d(t)$, between the wall and the rear of the chair varies sinusoidally with time t. At $t = 1$ s, the chair is closest to the wall and $d(1) = 18$ cm. At $t = 1.75$ s, the chair is farthest from the wall and $d(1.75) = 34$ cm.

a) What is the period of the function, and what does it represent in this situation?
b) How far is the chair from the wall when no one is rocking in it?
c) If Meagan rocks back and forth 40 times only, what is the domain of the function?
d) What is the range of the function in part (c)?
e) What is the amplitude of the function, and what does it represent in this situation?
f) What is the equation of the sinusoidal function?
g) What is the distance between the wall and the chair at $t = 8$ s?



a) Half period = 0.75
$\Rightarrow$ period = 1.5
Represents time to rock back and forth.

b) Axis $= \dfrac{34+18}{2} = 26$cm

c) $40(1.5) = 60$
$\Rightarrow D = \{t \in \mathbb{R} \mid 0 \le t \le 60\}$

d) $R = \{d \in \mathbb{R} \mid 18 \le d \le 34\}$

e) Amplitude $= \dfrac{34-18}{2}$
$= 8$cm
Represents distance chair rocks from resting position.

f) $d(t) = 8\cos(240(x - 0.25)) + 26$

$k = \dfrac{360}{1.5} = 240$

g) $d(8) = 8\cos(240(8 - 0.25)) + 26$
$= 30$cm