Review

1. Topics:

- Properties of Periodic Functions
- Sine Curve
- Cosine Curve
- Transformations of Sine and Cosine
- Modelling Trig Equations
- Solving Linear Trig Equations
- Solving Equations where $k \neq 1$
- Problem Solving

2. Questions

Nelson Page 404 #s 1 - 4, 5 (use desmos), 6 & 8 - 13

Vocabulary

Periodic function: a function whose graph repeats at regular intervals; the y-values in the table of values show a repetitive pattern when the x-values change by the same amount.

Period: the change in the independent variable (typically x) corresponding to one cycle.

Cycle: the cycle of a periodic function is a portion of the graph that repeats

Trough: the minimum point on a graph

Peak: the maximum point on a graph

Vocabulary

Axis of the curve: the equation of the horizontal line halfway between the maximum and the minimum. This is often called the "equation of the axis". It is determined by:

$$y = \frac{\text{max value} + \text{min value}}{2}$$

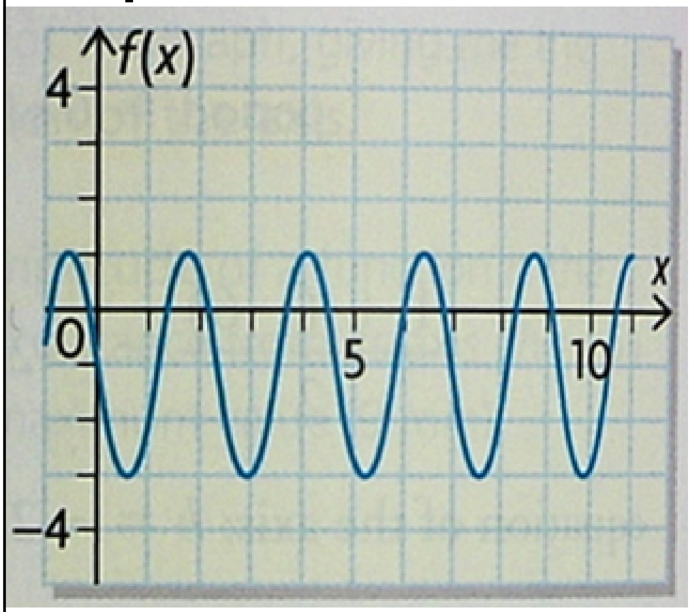
AXIS
$\Rightarrow$ ADD

Amplitude: half of the difference between the maximum and the minimum values; it is also the vertical distance from the function's axis to the max or min values. It is determined by:

$$\text{amplitude} = \frac{\text{max value} - \text{min value}}{2}$$

AMPLITUDE
$\Rightarrow$ SUBTRACT

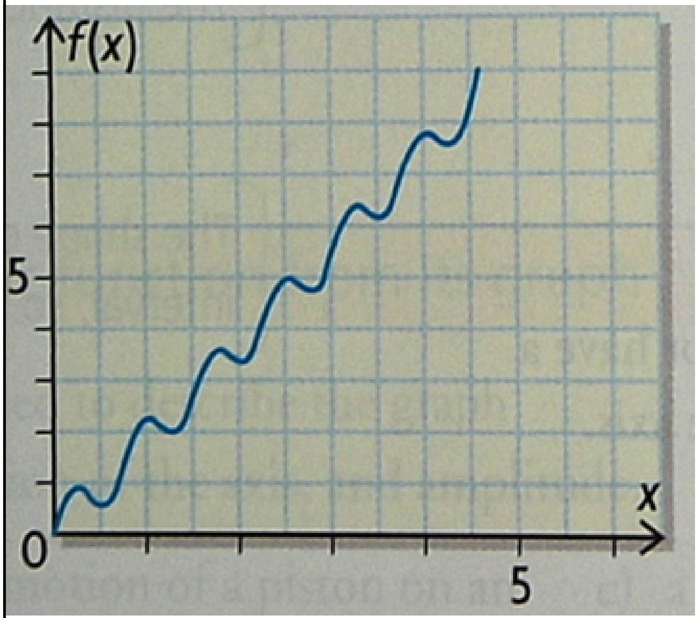
Graph # 2



Periodic?

Yes

Graph # 3



Periodic?

No

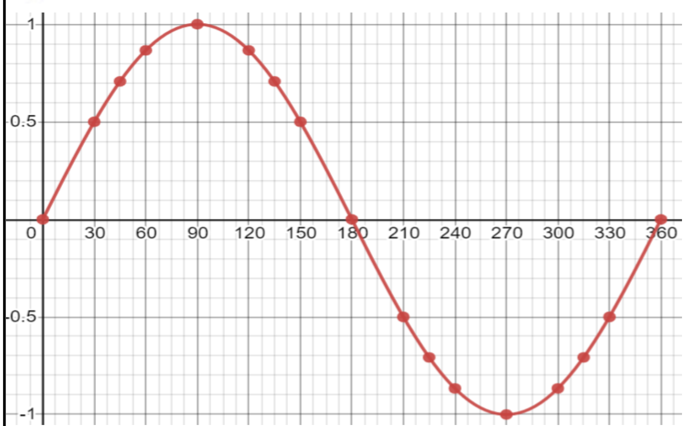
Sine and Cosine Curves

Name:____________________ Date:________________

1. Fill in the table below.

θ	0°	30°	45°	60°	90°	120°	135°	150°	180°
$\sin\theta$	0	0.5	0.707	0.866	1.0	0.866	0.707	0.5	0

210°	225°	240°	270°	300°	315°	330°	360°
-0.5	-0.707	-0.866	-1.0	-0.866	-0.707	-0.5	0

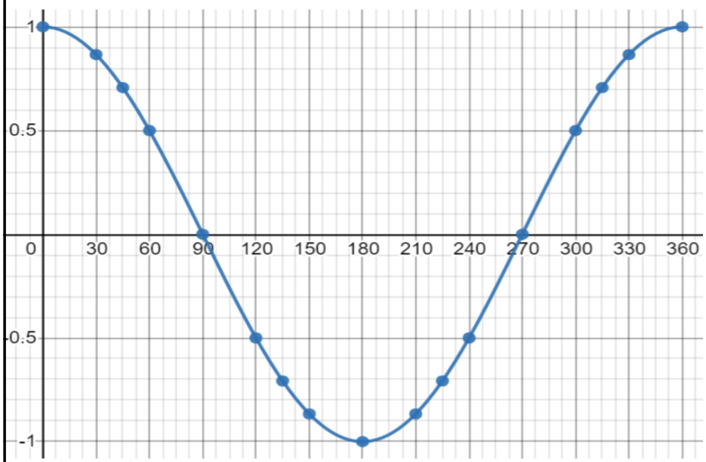


$y = \sin\theta$	
Maximum	1
Minimum	-1
Axis of Curve	y = 0
Amplitude	1
Period	360°
x-intercept(s)	0, 180, 360 (180n)
y-intercept	0

2. Fill in the table below.

θ	0°	30°	45°	60°	90°	120°	135°	150°	180°
$\cos\theta$	1.0	0.866	0.707	0.5	0	-0.5	-0.707	-0.866	-1

210°	225°	240°	270°	300°	315°	330°	360°
-0.866	-0.707	-0.5	0	0.5	0.707	0.866	1.0



$y = \cos\theta$	
Maximum	1
Minimum	-1
Axis of Curve	$y = 0$
Amplitude	1
Period	360°
x-intercept(s)	90, 270 (90+180n)
y-intercept	1

To summarise....

Trigonometric transformations follow the same rules as for polynomial functions

$y = a\sin(k(x - d)) + c$ and $y = a\cos(k(x - d)) + c$

where

a = vertical stretch/compression/reflection in x-axis
k = horizontal stretch/compression/reflection in y-axis
d = horizontal translation (known as a PHASE SHIFT for trig functions)
c = vertical translation

REMEMBER - To express "d" correctly, we must factor out the value of k if necessary

For example $y = 3\sin(4x + 80) - 1$

$y = 3\sin(4(x + 20)) - 1 \longrightarrow d = -20$, NOT -80

The key points on a sine or cosine curve are where
x = 0, 90, 180, 270 and 360

We can perform our transformations to each of these key points to see what one transformed cycle will now look like.

Horizontal transformations:

x-value ⟶ ÷ k ⟶ + d ⟶ New x-value

Vertical transformations:

y-value ⟶ x a ⟶ + c ⟶ New y-value

Warm Up:
State the transformations applied to the sine function.

$$f(x) = -2\sin(x - 60) + 3$$

Reflected in the x-axis (a is neg)
VS by a factor of 2 (|a| > 1)
HT right 60 (d = 60)
VT up 3 (c = 3)

Complete the chart below for each equation:

	y = cos(x)	y = 2cos(4x) + 1
max value	1	3
min value	-1	-1
axis of curve	y = 0	y = 1
amplitude	1	2
period	360°	90°
# of cycles in 360°	1	4
"a"	1	2
"k"	1	4
"d"	0	0
"c"	0	1

Any connections?

$k = \frac{360}{\text{Period}}$ $\left[\text{Period} = \frac{360}{k}\right]$

Axis of curve $\Rightarrow y = c$

Amplitude $\Rightarrow a$

Max $= c + |a|$

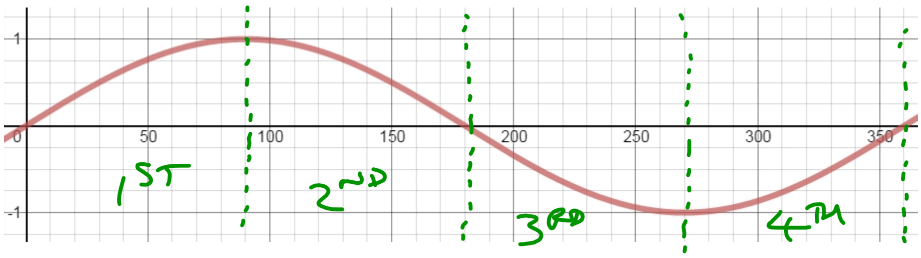
Min $= c - |a|$

positive "version" of a

Understanding the Symmetry of a Sinusoidal Curve

To be able to model using a sinusoidal curve we need to understand the symmetry to be able to draw out patterns.

Let's examine the sine function.



Important Points

Starts at: (0, 0)

First quarter: (90, 1)

Halfway: (180, 0)

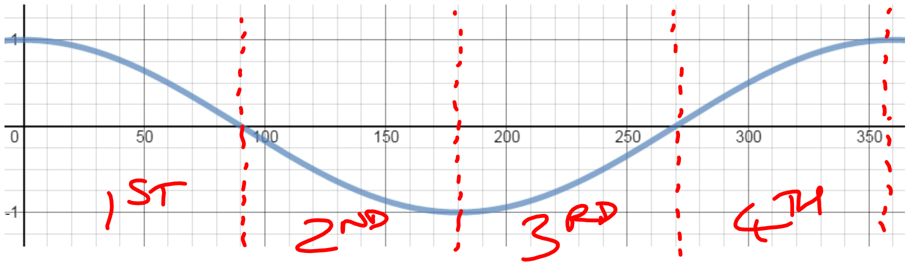
Third quarter: (270, -1)

Ends at: (360, 0)

Understanding the Symmetry of a Sinusoidal Curve

To be able to model using a sinusoidal curve we need to understand the symmetry to be able to draw out patterns.

Let's examine the cosine function.



Important Points

Starts at: (0, 1)

First quarter: (90, 0)

Halfway: (180, −1)

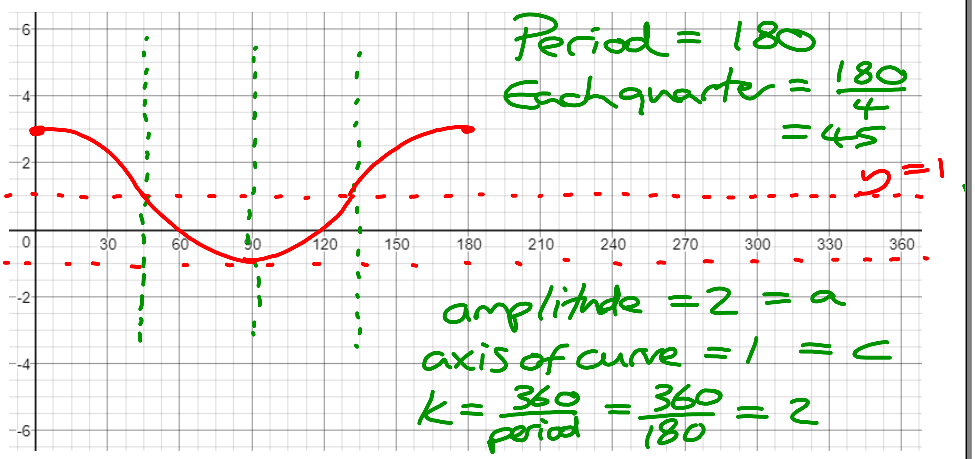
Third quarter: (270, 0)

Ends at: (360, 1)

Example

A sinusoidal function has an amplitude of 2 units, a period of 180°, and a max at (0,3). Represent the function with two different equations.

1. Sketch the curve.



2. Which function starts at a max? cosine

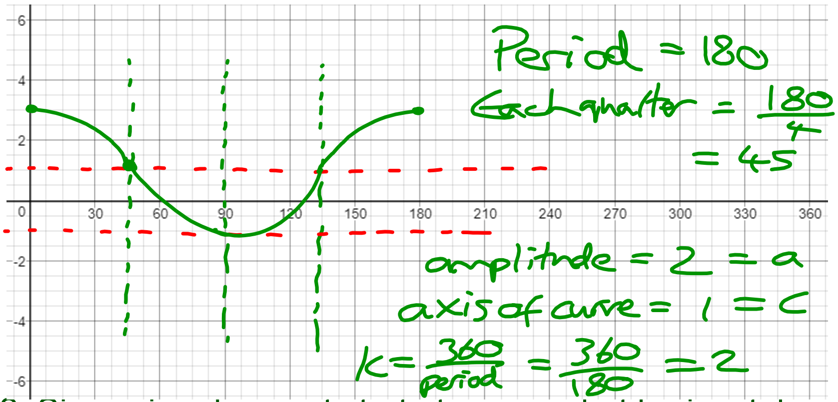
$y = a\cos(k(x-d))+c$

$\Rightarrow y = 2\cos(2x)+1$

Example

A sinusoidal function has an amplitude of 2 units, a period of 180°, and a max at (0,3). Represent the function with two different equations.

1. Sketch the curve.



2. Since sine does not start at a max what horizontal shift has taken place?

$y = a\sin(k(x-d)) + c$

$y = -2\sin(2(x-45)) + 1$

$-a$ because function decreases when $x = 45$

Example

Determine the equation that models this data:

x	0	30	60	90	120	150	180
y	3	2	1	2	3	2	1

What is the max? 3 $k = \frac{360}{\text{period}} = \frac{360}{120} = 3$

What is the min? 1

How long does it take for one cycle? 120 (Max → Max)

Where is the equation of the axis? $y = 2$

Do we start at the axis or at a max/min? Max ⇒ cos

$\Rightarrow y = \cos(3x) + 2$

Example

Determine the equation that models this data:

x	-180	0	180	360	540	720	900
y	17	13	17	21	17	13	17

What is the max? 21 $k = \frac{360}{720} = \frac{1}{2}$

What is the min? 13

How long does it take for one cycle? 720 (Min ⇒ Min)

Where is the equation of the axis? $y = 17$

Do we start at the axis or at a max/min? Min ⇒ $-\cos$

$$y = -4\cos(0.5x) + 17$$

$$\text{amp} = \frac{21-13}{2} = 4$$

Steps to follow when modelling:

1. Determine the max and min

2. Determine the axis of curve: $y = \frac{\text{max} + \text{min}}{2}$

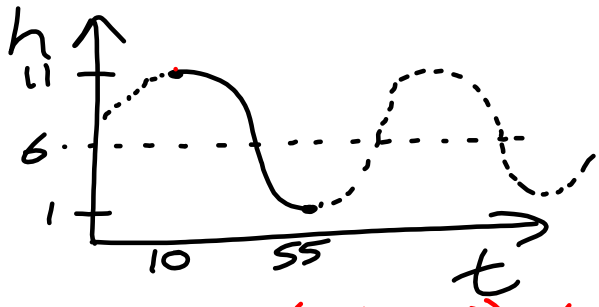
3. Determine the amplitude: $\frac{\text{max} - \text{min}}{2}$

4. Determine how many cycles are in 360 to find k. $\frac{360}{\text{period}}$

5. Decide which model
   Does the graph start at a max/min? ⟶ cosine
   Does the graph start at the axis? ⟶ sine
   Neither? ⟶ Your choice - you have to determine the phase shift!

Example

A group of students are tracking their friend, Bunter, on a Ferris Wheel. They know that Bunter reaches the maximum height of 11 metres at 10 seconds then reaches the minimum height above the ground of 1 metre at 55 seconds. Find the equation to model Bunter's height above the ground as he travels on the Ferris Wheel.



Max = 11

Min = 1

Axis of curve $= \frac{11+1}{2} = \frac{12}{2} = 6$

$y = 6$

Amplitude $= \frac{11-1}{2} = \frac{10}{2} = 5$

$\frac{1}{2}$ cycle = 55 − 10 = 45 seconds

$\Rightarrow$ 1 cycle = 45×2 = 90s

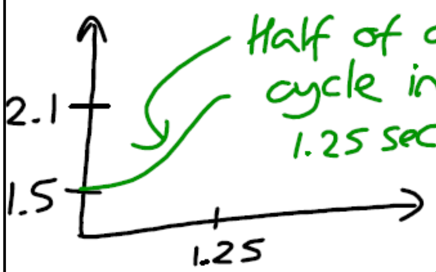
$k = \frac{360}{\text{period}} = \frac{360}{90} = 4$

$h = 5\cos(4(x-10)) + 6$

Example

Smudger is floating on an inner tube in a wave pool. He is1.5 metres from the bottom of the pool when he is at the trough of the wave. A stop watch starts timing at this point. In 1.25 seconds, he is on the peak of the wave, 2.1 metres from the bottom of the pool.

a) Determine the equation of the function that expresses Smudger's distance from the bottom of the pool in terms of time.



Half of a cycle in 1.25 sec $\Rightarrow$ period = 2(1.25) = 2.5

$k = \frac{360}{2.5} = 144$

Max = 2.1, Min = 1.5

Axis $= \frac{2.1+1.5}{2} = 1.8$

Amplitude $= \frac{2.1-1.5}{2} = 0.3$

Starts at a min so we can use −cos with no phase shift

$\Rightarrow y = -0.3\cos(144x) + 1.8$

b) How far above the bottom of the pool is Smudger after 4 seconds?

Sub in $x = 4$

$\Rightarrow -0.3\cos(144(4)) + 1.8$

$= 2.04\text{ m}$

c) If data was collected for a total of 40 seconds, how many complete cycles would have occurred?

Period $= 2.5$

$\Rightarrow$ # of cycles $= \frac{40}{2.5} = 16$

d) If the period of the function changes to 3 seconds, what is the equation of the new function?

New $k = \frac{360}{3} = 120$

Period affects nothing else

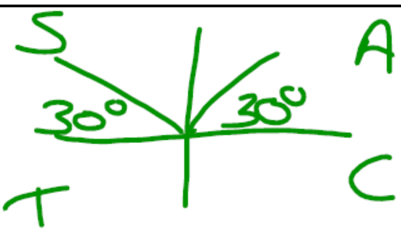
$\Rightarrow y = -0.3\cos(120x) + 1.8$

How do transformations affect the number of solutions?

Since there are 3 cycles between 0 and 360 for sin(3x) there must be _3_ times the number of solutions as there are for sin(x).

We need to find the solutions for the first cycle and then find the other solutions based on the __period____ of the cycle.

Algebraically



$\sin 3x = \frac{1}{2}$

$3x = \sin^{-1}\left(\frac{1}{2}\right)$

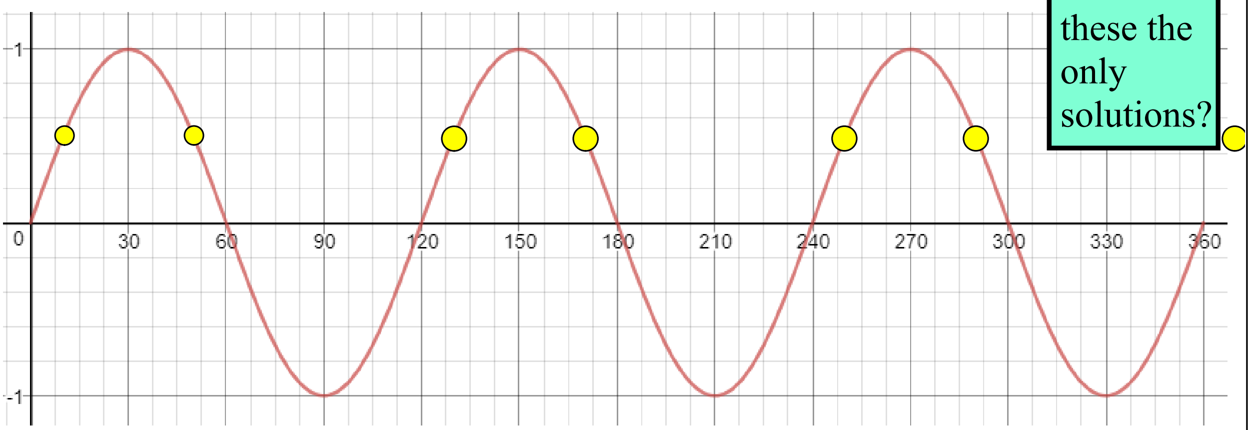
$\frac{3x}{3} = \frac{30°}{3}$ OR $\frac{3x}{3} = \frac{150°}{3}$

$x = 10°$ $x = 50°$

Solving Trigonometric Equations

Example

Where does the graph of $y = \sin(3x)$ have a y-value of ½ for $0 \leq x \leq 360°$?



But are these the only solutions?

Finding all of the solutions

Since the period is 120° we know the y-values repeat every 120°. So to get the next solutions we need to add 120° to the solutions we have.

$\boxed{x = 10}$ or $\boxed{x = 50}$

Recall

$\text{Period} = \frac{360}{k}$

$x = 10 + 120$
$x = 130^\circ$

$x = 130 + 120$
$x = 250^\circ$

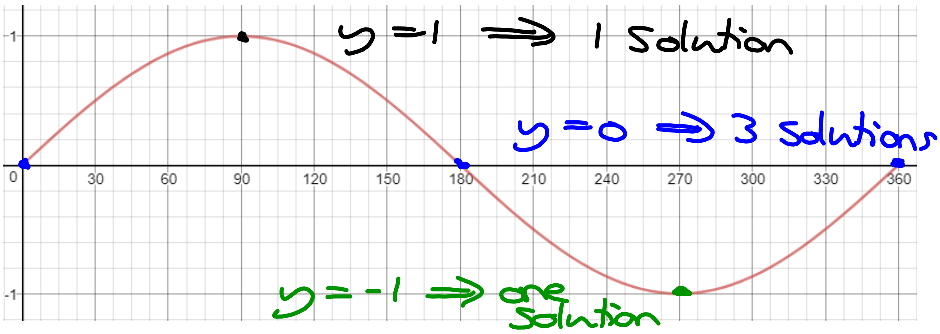
$x = 50 + 120$
$x = 170^\circ$

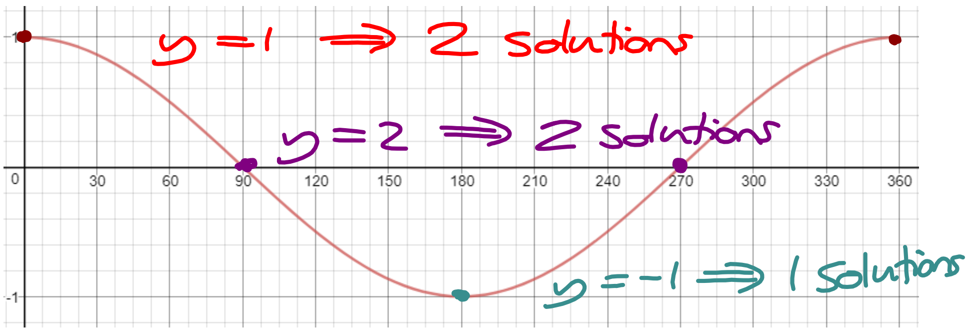
$x = 170 + 120$
$x = 290^\circ$

Finding the number of solutions when sin or cos = ±1 or 0

When sin(x) or cos(x) = ±1 or 0 we get a little bit of a different situation.

Look at their graphs, do you see what the difference may be?





Example

Find all solutions for $0 \leq x \leq 360$:

a) $\sin(2x) = 1$

$2x = \sin^{-1}(1)$

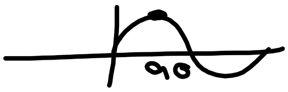
$\frac{2x}{2} = \frac{90}{2}$

$x = 45^\circ$

Add on period of $\frac{360}{2} = 180$

$x = 45 + 180$

$x = 225^\circ$



b) $\cos(\frac{x}{2}) = 0$

$\frac{x}{2} = \cos^{-1}(0)$

$\frac{x}{2} = 90$ OR $\frac{x}{2} = 270$

$x = 180$ or $x = 540$

Add on period of $\frac{360}{0.5} = 720$

Both are TOO BIG

$\Rightarrow x = 180$

(only one solution)

Steps to Solving a Trig Equation

1. SAMDEB - isolate for the trig function
2. Inverse trig function
3. CAST rule
4. Solve for the variable
5. Apply the period (if necessary)

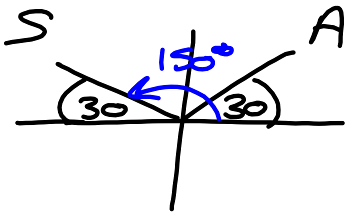
Example

Solve: $2\sin(3x) - 1 = 0$

$$\frac{2\sin(3x)}{2} = \frac{1}{2}$$

$$\sin(3x) = \frac{1}{2}$$

$$3x = \sin^{-1}\left(\frac{1}{2}\right)$$



$$\frac{3x}{x} = \frac{30^\circ}{3}$$

$$x = 10$$

$$\frac{3x}{3} = \frac{150^\circ}{3}$$

$$x = 50$$

$$\text{Period} = \frac{360}{k} = \frac{360}{3} = 120$$

$\Rightarrow$ $x = 10 + 120$

$x = 130$

$x = 50 + 120$

$x = 170$

$\Rightarrow$ $x = 130 + 120$

$x = 250$

$x = 170 + 120$

$x = 290$