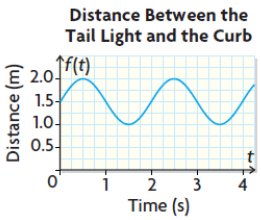
Solutions

1. The load on a trailer has shifted, causing the rear end of the trailer to swing left and right. The distance from one of the tail lights on the trailer to the curb varies sinusoidally with time. The graph models this behaviour.



a) What is the equation of the axis of the function, and what does it represent in this situation?
b) What is the amplitude of the function, and what does it represent in this situation?
c) What is the period of the function, and what does it represent in this situation?
d) Determine the equation and the range of the sinusoidal function.
e) What are the domain and range of the function in terms of the situation?
f) How far is the tail light from the curve at $t = 3.2$ s?

a) $d = \frac{\max + \min}{2}$

$d = \frac{2.0+1.0}{2}$

$d = \frac{3}{2}$

$d = 1.5$

Represents the distance of the tail light from the curb.

b) amplitude $= \frac{\max - \min}{2}$

$= \frac{2.0-1.0}{2}$

$= 0.5$m

Represents the distance it moves from to the left or right from the centre

c) Period = 2 seconds
Time taken for one cycle

d) $d = 0.5\sin(180t) + 1.5$

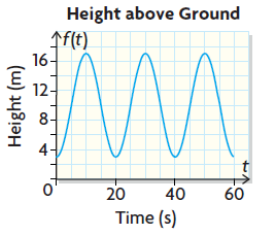
$R = \{d \in \mathbb{R} \mid 1 \leq d \leq 2\}$

e) Domain is time
Range is distance of the light from the curb.

f) $d = 0.5\sin(180 \times 3.2) + 1.5$

$d = 1.2$m

2. Don Quixote, a fictional character in a Spanish novel, attacked windmills because he thought they were giants. At one point, he got snagged by one of the blades and was hoisted into the air. The graph shows his height above ground in terms of time.



a) What is the equation of the axis of the function, and what does it represent in this situation?
b) What is the amplitude of the function, and what does it represent in this situation?
c) What is the period of the function, and what does it represent in this situation?
d) If Don Quixote remains snagged for seven complete cycles, determine the domain and range of the function.
e) Determine the equation of the sinusoidal function.
f) If the wind speed decreased, how would that affect the graph of the sinusoidal function?

a) $d = \frac{17+3}{2}$

$d = 10$

Represents the height of the axle.

b) amplitude $= \frac{17-3}{2}$

$= 7m$

Represents the length of the blades.

c) Period = 20 seconds

Represents the time for one revolution.

d) $D = \{t \in \mathbb{R} \mid 0 \le t \le 140\}$

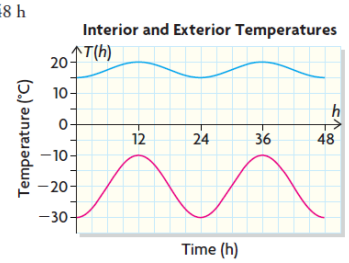
$R = \{d \in \mathbb{R} \mid 3 \le d \le 17\}$

e) $d = -7\cos(18t) + 10$

amplitude; $\frac{360}{\text{period}}$; axis of curve

f) It would be horizontally stretched $\Rightarrow$ period increases

4. The interior and exterior temperatures of an igloo were recorded over a 48 h period. The data were collected and plotted, and two curves were drawn through the appropriate points.



a) How are these curves similar? Explain how each of them might be related to this situation.
b) Describe the domain and range of each curve.
c) Assuming that the curves can be represented by sinusoidal functions, determine the equation of each function.

a) Both sinusoidal.
Same periods (24 hours)
Same times for max and min values.
Less change in temperature (blue) $\Rightarrow$ indoors
More change in temperature (red) $\Rightarrow$ outdoors

b) $D = \{t \in \mathbb{R} \mid 0 \le t \le 48\}$ $D = \{t \in \mathbb{R} \mid 0 \le t \le 48\}$

$R = \{T \in \mathbb{R} \mid 15 \le T \le 20\}$ $R = \{T \in \mathbb{R} \mid -30 \le T \le -10\}$

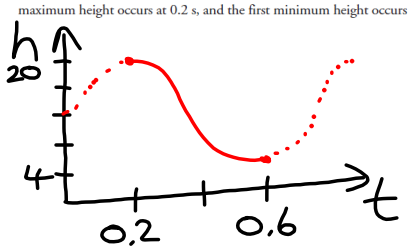
c) Periods both 24 $\Rightarrow k = \frac{360}{24} = 15$

$T = -2.5\cos(15t) + 17.5$

$T = -10\cos(15t) - 20$

8. Candice is holding onto the end of a spring that is attached to a lead ball. As she moves her hand slightly up and down, the ball moves up and down. With a little concentration, she can repeatedly get the ball to reach a maximum height of 20 cm and a minimum height of 4 cm from the top of a surface. The first maximum height occurs at 0.2 s, and the first minimum height occurs at 0.6 s.

a) Determine the equation of the sinusoidal function that represents the height of the lead ball in terms of time.
b) Determine the domain and range of the function.
c) What is the equation of the axis, and what does it represent in this situation?
d) What is the height of the lead ball at 1.3 s?



Max = 20 cm
Min = 4 cm
Axis = $\frac{20+4}{2}$ = 12
Amplitude $\frac{20-4}{2}$ = 8cm

Period = 2(0.4)
= 0.8

$\Rightarrow k = \frac{360}{0.8} = 450$

$h = 8\cos(450(x-0.2)) + 12$

phase shift, axis, amplitude

b) $D = \{t \in \mathbb{R}\}$
$R = \{h \in \mathbb{R} \mid 4 \le h \le 20\}$

c) Equation of axis
$h = 12$
Represents the resting height of the lead ball.

d) $h(1.3) = 8\cos(450(1.3-0.2)) + 12$
$h = 6.3$cm