Solving Trigonometric Equations Cont'd...

Nelson Page 398 #s 1, 2, 4 & 8

Nov 4-10:28 AM

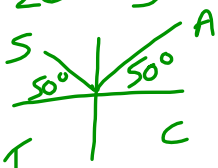
Warm Up

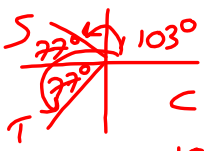
Find all possible solutions for $0 \leq \theta \leq 360^{\circ}$.

a) $\sin(2\theta) = 0.7629$ b) $\cos(2\theta) = -0.2315$

$2\theta = \sin^{-1}(0.7629)$ $2\theta = \cos^{-1}(-0.2315)$

$2\theta = 50^{\circ}$ $2\theta = 130^{\circ}$ $2\theta = 103^{\circ}$ $2\theta = 257^{\circ}$





$\frac{2\theta}{2} = \frac{50}{2}$ $\frac{2\theta}{2} = \frac{130}{2}$ $\frac{2\theta}{2} = \frac{103}{2}$ $\frac{2\theta}{2} = \frac{257}{2}$

$\theta = 25^{\circ}$ $\theta = 65^{\circ}$ $\theta = 51.5^{\circ}$ $\theta = 128.5^{\circ}$

$k = 2 \Rightarrow$ Period $= 180$ $k = 2 \Rightarrow$ Period $= 180$

$\Rightarrow \theta = 25 + 180$ $\theta = 65 + 180$ $\Rightarrow \theta = 51.5 + 180$

$\theta = 205^{\circ}$ $\theta = 245^{\circ}$ $= 231.5^{\circ}$

$\theta = 128.5 + 180$

$= 308.5^{\circ}$

Apr 30-10:13

Solving Sine and Cosine Functions

Just like when we solve a linear equation we need to keep working backwards until we have isolated the variable.

We will do BEDMAS backwards with the trig ratio being like a bracket!

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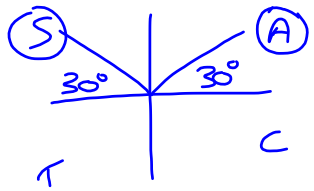
Example

Solve: $2\sin(3x) - 1 = 0$

SAMDER



$\frac{2\sin(3x)}{2} = \frac{1}{2}$

$\sin(3x) = \frac{1}{2}$

$3x = \sin^{-1}\left(\frac{1}{2}\right)$

$3x = 30°$

$3x = 180 - 30$

$3x = 150°$

$\frac{3x}{3} = \frac{30}{3}$

$\frac{3x}{3} = \frac{150}{3}$

$x = 10°$

$x = 50°$

$k = 3 \Rightarrow$ Period $= \frac{360}{3} = 120°$

↳ 3 pairs of solutions

$\Rightarrow x = 10 + 120$

$x = 50 + 120$

$x = 130°$

$x = 170°$

$\Rightarrow x = 130 + 120$

$x = 170 + 120$

$x = 250°$

$x = 290°$

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Example - Try this one on your own

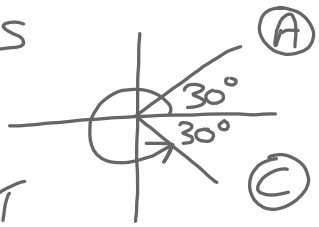
Solve: $2\cos(2x) - \sqrt{3} = 0$

SAMDER

$\frac{2\cos(2x)}{2} = \frac{\sqrt{3}}{2}$

$\cos(2x) = \frac{\sqrt{3}}{2}$

$2x = \cos^{-1}\left(\frac{\sqrt{3}}{2}\right)$



$2x = 30°$

$2x = 360 - 30$

$= 330°$

$\frac{2x}{2} = \frac{30}{2}$

$x = 15°$

$\frac{2x}{2} = \frac{330}{2}$

$x = 165°$

$k = 2 \Rightarrow$ Period $= 180$ $\left[\frac{360}{k}\right]$

$\Rightarrow x = 15 + 180$

$x = 195°$

$x = 165 + 180$

$x = 345°$

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Example - Try this one on your own

Solve: $2\cos(2x - 30) + 1 = 0$

SAMDER

$\frac{2\cos(2x-30)}{2} = \frac{-1}{2}$

$\cos(2x - 30) = -\frac{1}{2}$

$2x - 30 = \cos^{-1}\left(-\frac{1}{2}\right)$

$2x - 30 = 180 - 60$

$2x - 30 = 120$

$\frac{2x}{2} = \frac{150}{2}$

$x = 75°$

$2x - 30 = 180 + 60$

$2x - 30 = 240$

$\frac{2x}{2} = \frac{270}{2}$

$x = 135°$

$k = 2 \Rightarrow$ Period $= 180°$ $\left[\frac{360}{k}\right]$

$\Rightarrow x = 75 + 180$

$x = 255°$

$x = 135 + 180$

$x = 315°$

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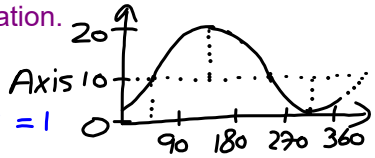
Example

Consider a function that has a max at 20, a min at 0, a period of 360° and increases through the axis of curve at x = 45°.

Determine a model for this equation.



$Amp(a) = \frac{20-0}{2} = 10$

$Axis(c) = 10$

$Period = 360 \Rightarrow k = \frac{360}{360} = 1$

Increases through $x = 45$ $(d) \Rightarrow y = 10\sin(x-45)+10$

When does this equation have a value of 18 for $0 \le x \le 360$?

$10\sin(x-45)+10 = 18$

$\frac{10\sin(x-45)}{10} = \frac{8}{10}$

$\sin(x-45) = 0.8$

$x-45 = \sin^{-1}(0.8)$

$x - 45 = 53$

$x = 98°$

SAMDES

$x - 45 = 180 - 53$

$x - 45 = 127$

$x = 172°$

Because $k=1$, we have no further solutions for $0 \le x \le 360°$.

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Example

A Ferris wheel has a radius of 7 m. The centre of the wheel is 8 m above the ground. The Ferris wheel rotates at a constant speed of 15°/s. The height above the ground of the only red seat can be modelled by the function $h(t) = 8 + 7\sin(15°t)$.

(a) Determine the height of the red seat at the start of the ride.

$h(0) = 8 + 7\sin(15(0)) = 8m$

(b) What is the maximum height of any seat?

$Max = c + |a| = 8 + |7| = 15m$

(c) When is the red seat at its maximum height during the first rotation?

$8 + 7\sin(15t) = 15$

$\frac{7\sin(15t)}{7} = \frac{7}{7}$

$\sin(15t) = 1$

$15t = \sin^{-1}(1)$

$\frac{15t}{15} = \frac{90}{15}$

$t = 6$ seconds

(d) What is the minimum height of any seat?

$Min = c - |a| = 8 - |7| = 1m$

(e) When is the red seat at its minimum height during the first rotation?

$8 + 7\sin(15t) = 1$

$\frac{7\sin(15t)}{7} = \frac{-7}{7}$

$\sin(15t) = -1$

$15t = \sin^{-1}(-1)$

$\frac{15t}{15} = \frac{270}{15}$

$t = 18$ seconds

(f) How long will it take for the red seat to complete two full rotations?

$Period = \frac{360}{k} = \frac{360}{15} = 24$ seconds

$\Rightarrow$ Two revs takes $2(24) = 48$ seconds

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Homework

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Mar 19-7:45 AM