Quadratic Transformations

Lesson objectives

- I understand and can identify a vertical stretch/compression and reflection
- I understand and can identify a horizontal translation
- I understand and can identify a vertical translation

1.1 Lesson objectives | Teachers' notes | Lesson notes

MHR Page 178 #s 1 – 3, 4aefh, 6 – 9, 13 & 15

Warm Up

Identify the vertex of each of the following parabolas.

$y = 3(x-1)^2 + 5$ $y = -2(x+3)^2 + 7$ $y = 4(x-2)^2 - 3$

The vertex is the point (h,k) when the equation is in the form:

$$y = a(x - h)^2 + k$$

Recall: h changes sign when it goes in or out of the bracket.

Vertex is (1,5) Vertex is (-3,7) Vertex is (2,-3)

Vertical Stretch / Compression / Reflection

$$y = ax^2$$

If |a| > 1 - then it is a vertical stretch by a factor of |a|
- it makes the parabola thinner

0 < |a| < 1 - then it is a vertical compression by a factor of |a|
- it makes the parabola wider

a < 0 - then there is a vertical reflection in the x-axis
- turns it upside down

|a| means to make it positive. It called the absolute value.

Vertical Translation

$$y = x^2 + k$$

If k > 0 - then it is a vertical translation up k units

k < 0 - then it is a vertical translation down k units

Horizontal Translation

$$y = (x - h)^2$$

** remember to change the sign of h when it comes out of the brackets **

If h > 0 - then it is a horizontal translation right h units
[looks like $(x - 4)^2$ ⟶ h = 4]

h < 0 - then it is a horizontal translation left h units
[looks like $(x + 4)^2$ ⟶ h = -4]

Example

Determine the transformations.

$y = 3(x-1)^2 + 5$

Short form

Vertical stretch by a factor of 3 — VS by a factor of 3
Horizontal translation 1 unit right — HT right 1
Vertical translation 5 units up — VT up 5

$y = -2(x+3)^2 + 7$

Vertical stretch by a factor of 2 — VS by a factor of 2
Reflection in the x-axis — Reflection in the x-axis
Horizontal translation 3 units left — HT left 3
Vertical translation 7 units up — VT up 7

$y = \frac{1}{3}(x-2)^2 - 3$

Vertical compression by a factor of 1/3 — VC by a factor of 1/3
Horizontal translation 2 units right — HT right 2
Vertical translation 3 units down — VT down 3

Graphing Using Transformations

When we apply transformations we understand that horizontal transformations affect the x-coordinate and vertical transformations affect the y-coordinate.

In other words:
a and k affect the y - (multiply then add)
h affects the x

Graphing

We can apply the transformations point by point or we can change the whole graph using the step pattern.

Example: List the transformations to $y = 2(x - 1)^2 + 2$ and identify the coordinates of the transformed points (1,1) and (3,9).

VS by a factor of 2
HT right 1
VT up 2

To apply transformations to points we do the following:

$x \longrightarrow x + h$

$y \longrightarrow a(y) + k$

So for the point (1,1)

$x = 1: \longrightarrow 1 + 1 = 2$

$y = 1: \longrightarrow 2(1) + 2 = 4$

(1,1) moves to (2,4)

So for the point (3,9)

$x = 3: \longrightarrow 3 + 1 = 4$

$y = 9: \longrightarrow 2(9) + 2 = 20$

(3,9) moves to (4,20)

Graphing Using the Step Pattern

Example: Graph the equation $y = 3(x + 1)^2 - 8$

Start by plotting the vertex (-1,-8)

Next multiply (1,3,5) by the "a" value

⟶ 3(1,3,5) = (3,9,15)

From the vertex move left one and then up 3, then from there left one and then up 9, and then from there left one and up 15.

Repeat the process but this time move right from the vertex and move right each time instead of left.

If these step numbers are negative then you would go down instead of up.