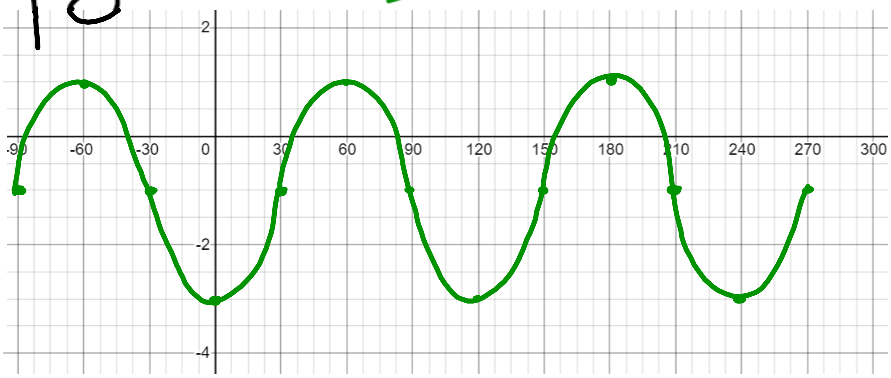
Warm Up $x \rightarrow \frac{x}{k} + d \; , \; y = ay + c$

Sketch the following graph for $0 \leq x \leq 360°$.

$$y = 2\sin(3(x + 90)) - 1$$

x	y
0	0
90	1
180	0
270	-1
360	0

New x	New y
-90	-1
-60	1
-30	-1
0	-3
30	-1



Solving Trigonometric Equations

Lesson objectives

- I can find the first set of solutions when solving an equation
- I can apply the period of the function to determine the number of cycles needed
- I can apply the period of the function to determine the solutions in the next cycle

1.1

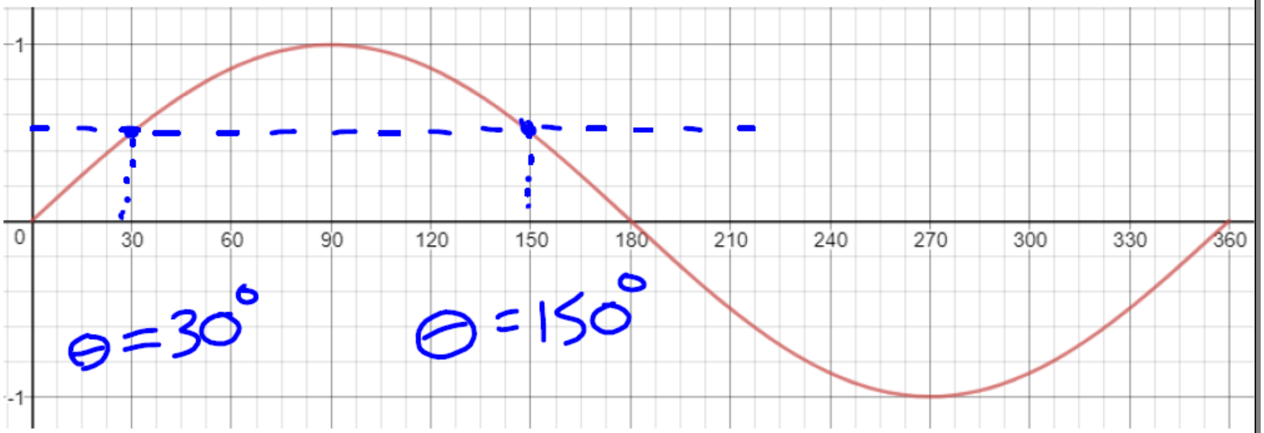
Lesson objectives | Teachers' notes | Lesson notes

Complete Questions from this Handout

Solving Trigonometric Equations

Example

Where does the graph of $y = \sin(x)$ have a y-value of ½ for $0 \leq \theta \leq 360^{\circ}$?



Solving Trigonometric Equations

Example

Where does the graph of $y = \sin(3x)$ have a y-value of ½ for $0 \leq x \leq 360^{\circ}$?

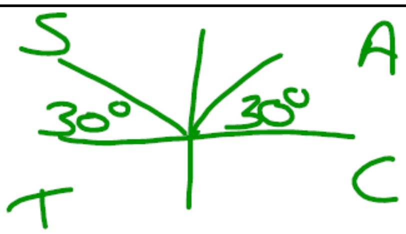
We have 6 solutions

How do transformations affect the number of solutions?

Since there are 3 cycles between 0 and 360 for sin(3x) there must be 3 times the number of solutions as there are for sin(x).

We need to find the solutions for the first cycle and then find the other solutions based on the length/period of the cycle.

Algebraically



$\sin 3x = \frac{1}{2}$

$3x = \sin^{-1}\left(\frac{1}{2}\right)$

$\frac{3x}{3} = \frac{30^\circ}{3}$ OR $\frac{3x}{3} = \frac{150^\circ}{3}$

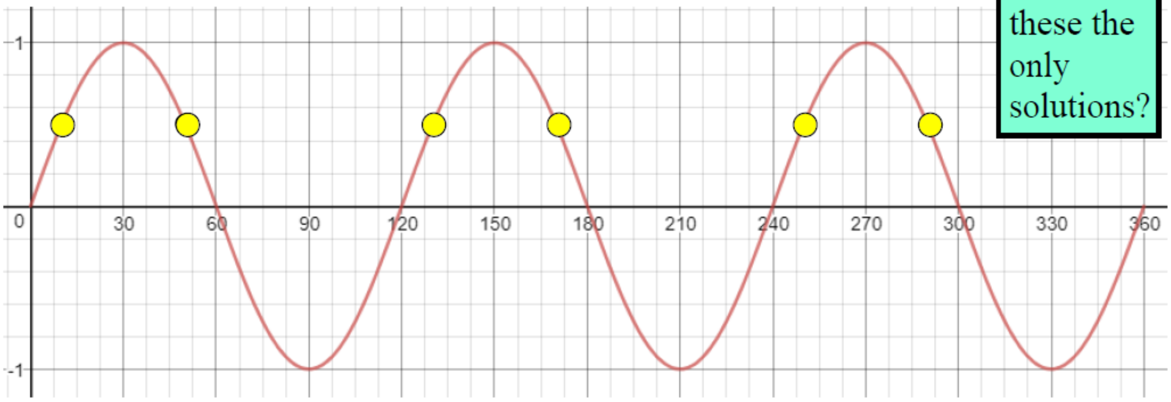
$x = 10^\circ$

$x = 50^\circ$

Solving Trigonometric Equations

Example

Where does the graph of y = sin(3x) have a y-value of ½ for $0 \leq x \leq 360^{o}$?



But are these the only solutions?

No! We need to add on the period.

Finding all of the solutions

Since the period is 120° we know the y-values repeat every 120°. So to get the next solutions we need to add 120° to the solutions we have.

x = 10 or x = 50

Recall

$\text{Period} = \frac{360}{k}$

$x = 10 + 120$

$x = 130^{\circ}$

$x = 130 + 120$

$x = 250^{\circ}$

$x = 50 + 120$

$x = 170^{\circ}$

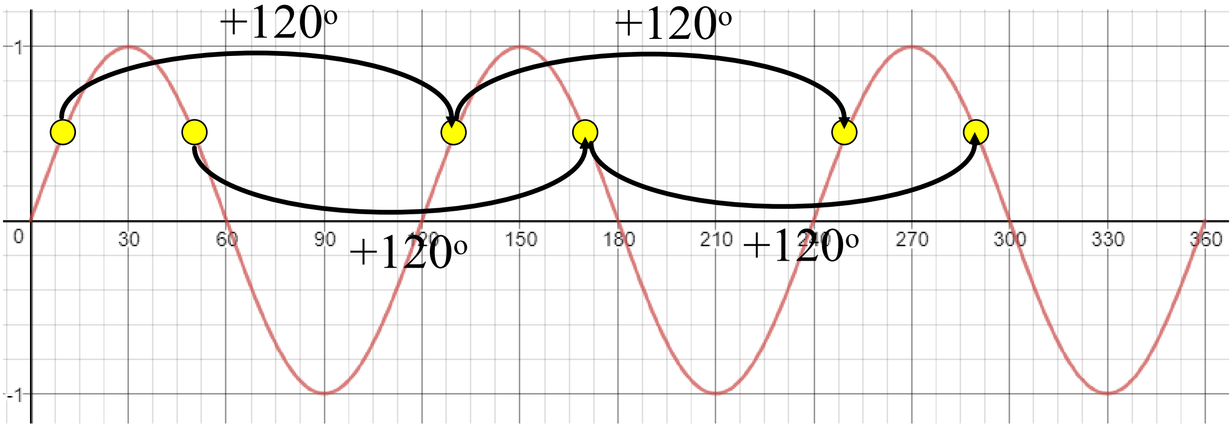
$x = 170 + 120$

$x = 290^{\circ}$

Solving Trigonometric Equations

Example

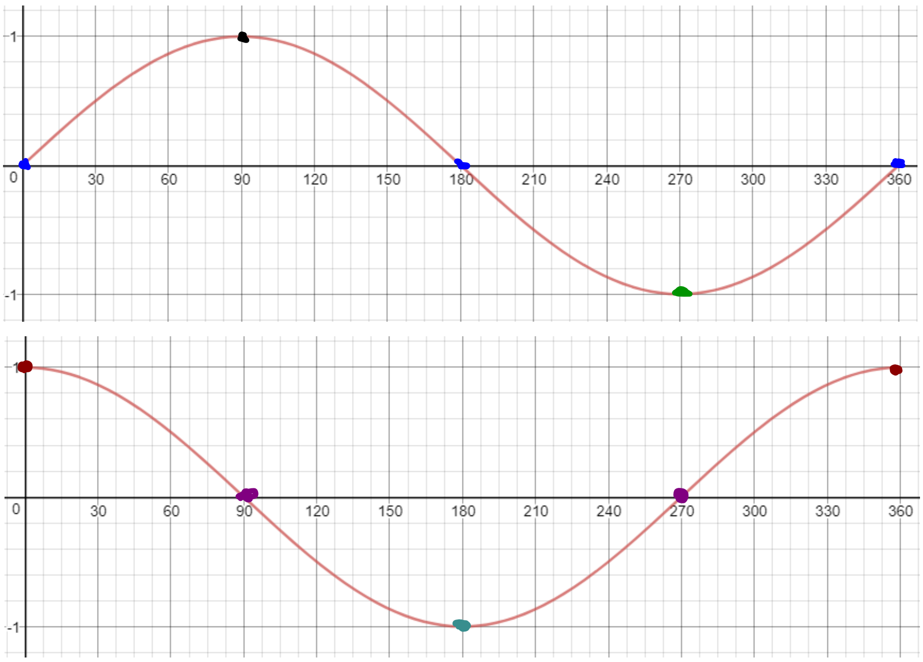
Where does the graph of y = sin(3x) have a y-value of ½ for $0 \leq x \leq 360^{o}$?



Finding the number of solutions when sin or cos = ±1 or 0

When sin(x) or cos(x) = ±1 or 0 we get a little bit of a different situation.

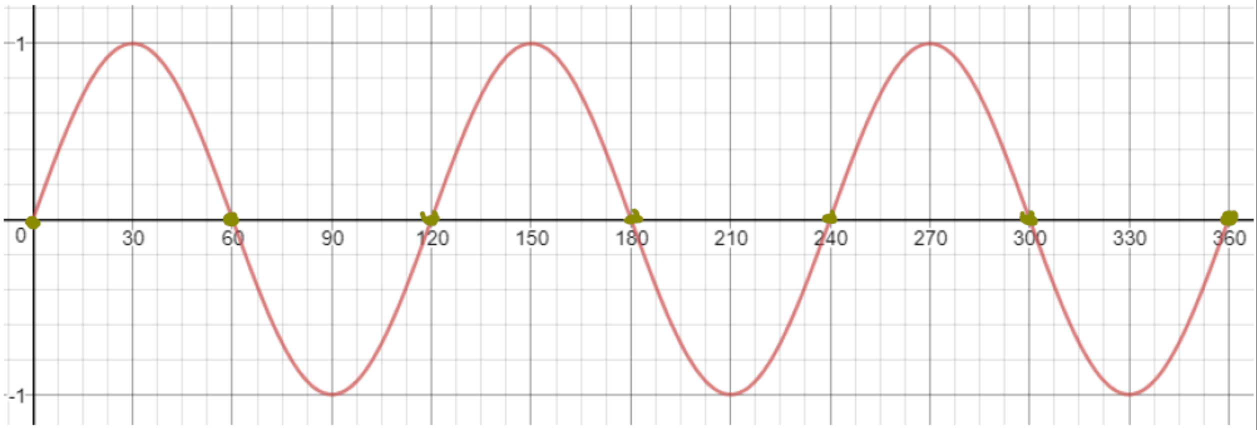
Look at their graphs, do you see what the difference may be?



Solving Trigonometric Equations

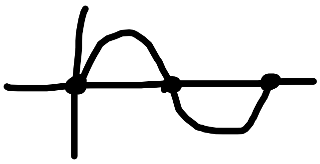
Example

Where does the graph of y = sin(3x) have a y-value of 0 for $0 \leq x \leq 360^{o}$?



There are 7 distinct solutions

Algebraically



sin3x = 0

$3x = \sin^{-1}(0)$

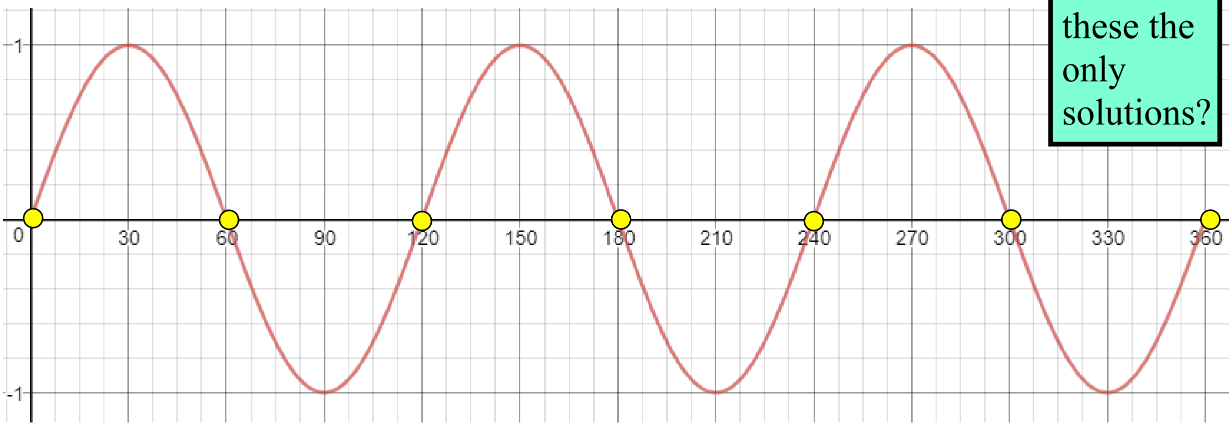
$\frac{3x}{3} = \frac{0}{3}$ OR $\frac{3x}{3} = \frac{180}{3}$ OR $\frac{3x}{3} = \frac{360}{3}$

$x = 0$ $\quad x = 60$ $\quad x = 120$

Solving Trigonometric Equations

Example

Where does the graph of y = sin(3x) have a y-value of 0 for $0 \leq x \leq 360^o$?



But are these the only solutions?

Finding all of the solutions

Since the period is 120° we know the y-values repeat every 120°. So to get the next solutions we need to add 120° to the solutions we have.

$\boxed{x = 0}$ or $\boxed{x = 60}$ or $\boxed{x = 120}$

$x = 0 + 120$

$x = 120^\circ$

REPEATED

$x = 120 + 120$

$x = 240^\circ$

REPEATED

$x = 60 + 120$

$x = 180^\circ$

$x = 180 + 120$

$x = 300^\circ$

$x = 120 + 120$

$x = 240^\circ$

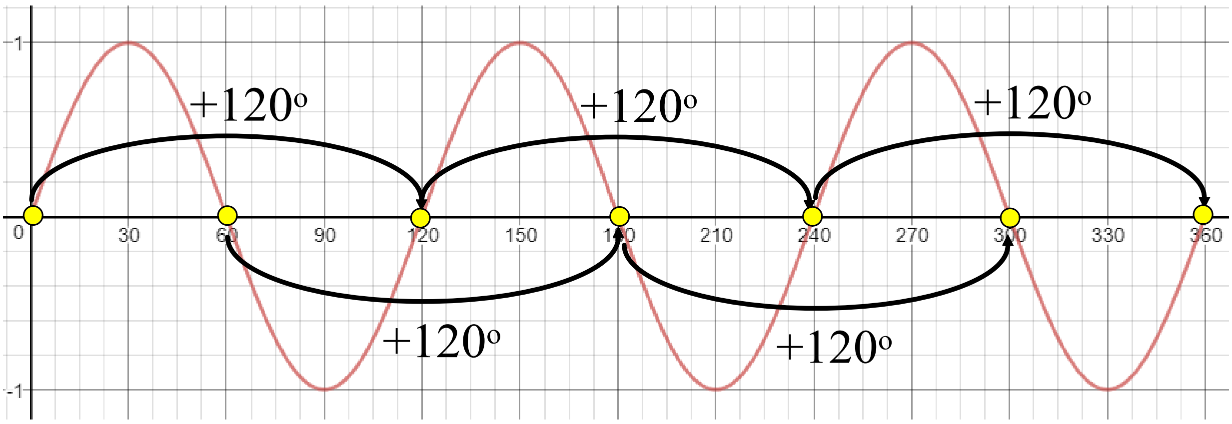
$x = 240 + 120$

$x = 360^\circ$

Solving Trigonometric Equations

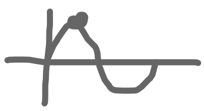
Example

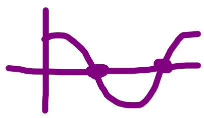
Where does the graph of y = sin(3x) have a y-value of 0 for $0 \leq x \leq 360^o$?



Example

Find all solutions for $0 \leq x \leq 360$:





a) sin(2x) = 1 b) $\cos(\frac{x}{2}) = 0$

a)

$2x = \sin^{-1}(1)$

$\frac{2x}{2} = \frac{90}{2}$ (only)

$x = 45^\circ$

Period $= \frac{360}{2} = 180^\circ$

$\Rightarrow x = 45 + 180$

$x = 225^\circ$

b)

$\frac{x}{2} = \cos^{-1}(0)$

$\frac{x}{2} = 90$ OR $\frac{x}{2} = 270$

$x = 180^\circ$ $x = 540$

Period $= \frac{360}{1/2} = 720^\circ$

If we add/subtract the period the answers are outside of $0 \leq x \leq 360$, so they are extraneous.

Homework

Solve the following for $0° \leq x \leq 360°$

1. $2\cos(3x) = 1$

2. $2\sin\left(\frac{x}{2}\right) = -1$

3. $4\sin(2x) - 3 = 0$

4. $3\sin(2x) = -3$

5. $4\cos(4x) - 1 = 3$

6. $4\sin(3x) - \sqrt{3} = \sqrt{3}$